Toulouse School of Economics, 2013-2014
Macroeconomics L3 Semester 1 – Franck Portier

Problem set 6: Sustainable growth
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Problem I – Natural resources in the Solow model.

Some factors of production are available in finite supply, typically natural resources (land, oil, etc...). Therefore, it is often argued that economic growth should eventually stop. Let us consider this view in a Solow model.

Let the production function be $Y = K^{\alpha}(AL)^{\beta}T^{1-\alpha-\beta}$ where K is capital, L labor, T land and A productivity (think of any variable X as being $X(t)$). Assume $\alpha > 0$, $\beta > 0$ and $\alpha + \beta < 1$. Factors evoke according to:

$$\begin{aligned} \dot{K} &= sY - \delta K \\ \dot{A} &= gA \\ \dot{L} &= nL \\ \dot{T} &= 0 \end{aligned}$$

1 – Discuss the model assumptions

2 – Does the economy have a unique and stable balanced growth path, where all variables grow at a constant but not necessarily similar rate)? If so, what are the growth rates? If not, why not?

3 – Given your answer to the previous question, does the fact that the stock of land is constant imply that permanent growth rate is not possible? Explain.

4 – Under which circumstance will $\frac{Y}{L}$ grow over time, as opposed to shrinking steadily? Give your answer in terms of α, β, n and g.

5 – The Malthusian view assesses that if resource are finite, then the economy will end in a no growth trap. Discuss that prediction in the light of this model.

Problem II – Simulation of the model of sustainable development

Take the model of slide 41 in section 4 of Lecture 6:

$$\begin{cases} S_{\tau+1} &= \left((1+r\Delta) - \left(\frac{r\Delta}{K}\right) S_{\tau} - \alpha\beta\Delta L_{\tau}\right) S_{\tau} \qquad (\mathcal{A}) \\ L_{\tau+1} &= \left((1-(b-d)\Delta) + \phi\alpha\beta\Delta S(t)\right)\right) L(t) \qquad (\mathcal{B}) \end{cases}$$

Assume the following values for parameters: $K = 12000$, $\Delta = 10$, $\alpha = .00001$, $\beta = 4$, $r = 0.04$, $b-d = -0.1$, $\phi = 4$, $S_0 = K$ and $L_0 = 40$.

1 – Use a spreadsheet to reproduce Figure 12 of lecture 6 slides. Print the figures and the spreadsheet with the computation.

2 – Increase r to 0.3 to reproduce Figure 13.

3 – Assume $S_0 = K = 4000$ to reproduce Figure 14.

4 – Propose one simulation with some changes (you should explain why throe changes are interesting) in parameters and/or initial conditions.

Problem III – The dynamics of a fish population

Suppose the growth rate of a population of wild fish is given by $\dot{x} = F(x) = 10x - 0.01x^2$ if the population is left undisturbed, were x is population size.

1 – What is the equilibrium natural population?

2 – What is the maximum sustained yield, and the corresponding population? Note that the maximum sustained yield is the maximum number of fish that can be caught per period while keeping the population of fish constant.

3 – Graph the population growth function. Which portions of this function correspond to biological overfishing (meaning that the population will tend to zero because the number of catch)?

Suppose now that the number of fish caught (harvest) is given by $h = 2Ex$, where E is the amount of fishing effort.

4 – Find the sustainable population x as a function of effort E.

5 – Find sustainable yield as a function of E.

6 – Graph the resulting Yield-Effort curve. Which portions of this curve correspond to biological overfishing?

Suppose now that the price of fish is $P = 10$, and total cost of fishing effort is $C(E) = 1000E$.

7 – Find the Total Revenue Product Curve.

8 – Suppose the fishery is managed by an owner with exclusive property rights. Find the level of effort E that maximizes the rent (revenue) from the fishery, and calculate that maximum rent.

9 – Under open access to the resource, any fisher can access the resource, so that the only equilibrium level of fishing is the one that drives the rent to zero. Find the level of effort that would occur under open access.

10 – Under open access, is this fishery suffering from economic overfishing (meaning that the rent is not maximum)?

11 – Under open access, is this fishery suffering from biological overfishing? Why or why not?