Put the barcode here **SOLUTION**

Exam

Instructions:

- *This is a 2h30 hours exam.*
- ***BE SURE TO PUT THE STICKER ON THE EXAM.***
- *There are a total of 100 points on the test.*
- *You have to answer all the questions.*
- ***Use ONLY the space provided for answering the questions.***
- ***You must write legibly.***

Part I *(5 points)*

Consider a production function for which output, y, is produced using labor, l, via the following production function (where $A > 0$ and $f > 0$):

$$y(l) = \max\{0, Al - f\}$$

In this production function, f represents a fixed cost of production.

For $l > f$, does this function exhibit increasing, decreasing, or constant returns to scale? Use the definition of returns to scale to explain your answer.

Consider a constant $\kappa > 1$. Then, for $l > \frac{f}{A}$:

$$y(\kappa l) = \max\{0, Al\kappa - f\} > \kappa \max\{0, Al - f\} = \kappa y(l),$$

which implies increasing returns to scale.
(If $l > f$ but happens to be less than $\frac{f}{A}$, then the production function may exhibit constant returns to scale. This wasn't really the point of the problem, so you weren't penalized if you didn't think of this possibility.)

PART 2 *(21 points)*
3 points for question 1; 6 points for question 3; 4 points each for questions 2, 4, and 5)

Consider an economy in which output is produced using capital, labor, and intermediate goods:

$$Y_t = \bar{A}\left(K_t^{\alpha} L_t^{1-\alpha}\right)^{1-\sigma} (X_t)^{\sigma} \tag{1}$$

Output is either consumed, saved (to produce capital next period), or used as an intermediate input:

$$Y_t = C_t + I_t + X_t \tag{2}$$

Capital grows according to the following rule:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

The population is constant across time:

$$L_t = \bar{L} \ \forall \ t \tag{4}$$

Assume that a constant fraction of output is invested

$$I_t = sY_t, \tag{5}$$

and that a constant fraction of output is used as an intermediate input

$$X_t = rY_t. \tag{6}$$

1 – Use lower case letters to write variables in per capita terms (for example, capital per capita at time t is $\frac{K_t}{L_t} = k_t$). Re-write Equations (1), (2), (3),(5), and (6) in per capita terms.

$y_t = \bar{A}\left(\frac{K_t^{\alpha} L_t^{1-\alpha}}{L_t}\right)^{1-\sigma}\left(\frac{X_t}{L_t}\right)^{\sigma} = \bar{A}\left(k_t\right)^{\alpha(1-\sigma)}\left(x_t\right)^{\sigma}$
$y_t = c_t + i_t + x_t$
$k_{t+1} = (1-\delta)\, k_t + i_t$
$i_t = sy_t$
$x_t = ry_t$

2 – Use "ss" to denote the steady state value of a particular variable (for example, let k_{ss} be the steady state capital-labor ratio.) Explain why $\delta k_{ss} = sy_{ss}$ (one sentence max). Using this equation, as well as the per- capita version of Equation (6), solve for y_{ss} in terms of $\bar{A}, r, s, \delta$, α, and σ.

Combining equations we know that: $k_{t+1} = (1-\delta)\,k_t + sy_t$.
At the steady state $k_{t+1} = k_t$ so that: $k_t = (1-\delta)\,k_t + sy_t$,, which implies $\delta k_t = sy_t \Rightarrow k_t = \frac{s}{\delta} y_t$
For the second part of the problem, we use the equation given to us: $x_t = ry_t$ P lugging stuff in and re-arranging:

$$\begin{aligned} y_{ss} &= \bar{A}\left(\frac{s}{\delta}y_{ss}\right)^{\alpha(1-\sigma)}(ry_{ss})^{\sigma} \\ &= \bar{A}s^{\alpha(1-\sigma)}r^{\sigma}\left(y_{ss}\right)^{\sigma+\alpha(1-\sigma)}\delta^{-\alpha(1-\sigma)} \\ \left(y_{ss}\right)^{(1-\sigma)(1-\alpha)} &= \bar{A}s^{\alpha(1-\sigma)}r^{\sigma}\delta^{-\alpha(1-\sigma)} \\ y_{ss} &= \bar{A}^{\frac{1}{(1-\sigma)(1-\alpha)}}s^{\frac{\alpha}{1-\alpha}}r^{\frac{\sigma}{(1-\sigma)(1-\alpha)}}\delta^{-\frac{\alpha}{1-\alpha}} \end{aligned}$$

3 – Using your answer to question 2, write c_{ss} in terms of $\bar{A}, r, s, \delta$, α, and σ. Then find the r and s which maximize c_{ss}. Use s_{gold} to denote the optimal saving rate, and r^* to denote the optimal intermediate-input utilization rate. For the rest of the problem, assume that $r = r^*$ and $s = s_{gold}$.

$c_{ss} = (1-r-s)\,y_{ss} = (1-r-s)\,\bar{A}^{\frac{1}{(1-\sigma)(1-\alpha)}}s^{\frac{\alpha}{1-\alpha}}r^{\frac{\sigma}{(1-\sigma)(1-\alpha)}}\delta^{-\frac{\alpha}{1-\alpha}}$. Take first order conditions of c_{ss}, first with respect to s, then with respect to r:

$$\begin{aligned} [s] \quad &: \quad \bar{A}^{\frac{1}{(1-\sigma)(1-\alpha)}}s^{\frac{\alpha}{1-\alpha}}r^{\frac{\sigma}{(1-\sigma)(1-\alpha)}}\delta^{-\frac{\alpha}{1-\alpha}}\left(-1+\frac{\alpha\,(1-r-s)}{1-\alpha}\frac{1}{s}\right)=0 \\ [r] \quad &: \quad \bar{A}^{\frac{1}{(1-\sigma)(1-\alpha)}}s^{\frac{\alpha}{1-\alpha}}r^{\frac{\sigma}{(1-\sigma)(1-\alpha)}}\delta^{-\frac{\alpha}{1-\alpha}}\left(-1+\frac{\sigma\,(1-r-s)}{(1-\sigma)\,(1-\alpha)}\frac{1}{r}\right)=0 \end{aligned}$$

Re-arranging gives $r = \sigma\frac{(1-r-s)}{(1-\sigma)(1-\alpha)}$ and $s = (1-r-s)\,\frac{\alpha}{(1-\alpha)}$ and therefore $r = s\frac{\sigma}{(1-\sigma)\alpha}$
Re-arranging some more:

$$\begin{aligned} s &= \left(1 - s\frac{\sigma}{(1-\sigma)\,\alpha} - s\frac{\alpha\,(1-\sigma)}{\alpha\,(1-\sigma)}\right)\frac{\alpha}{(1-\alpha)} \\ s\,(1-\alpha)\,(1-\sigma) &= \alpha\,(1-\sigma) - s\,[\sigma+\alpha-\alpha\sigma] \\ s\,[1-\alpha-\sigma+\alpha\sigma] &= \alpha\,(1-\sigma) - s\,[\sigma+\alpha-\alpha\sigma] \\ s &= \alpha\,(1-\sigma) \\ r &= \sigma \end{aligned}$$

4 – Plug your answer from question 3 into y_{ss} to determine the optimal steady-state output-labor ratio solely as a function of δ and the parameters of the production technology ($\bar{A}$, σ and α). Compute the elasticity of per capita GDP with respect to technology, $\varepsilon = \frac{\partial y_{ss}}{\partial \bar{A}}\frac{\bar{A}}{y_{ss}}$.

$$\mathbf{y}_{ss} = \bar{A}^{\frac{1}{(1-\sigma)(1-\alpha)}} \left(\alpha\left(1-\sigma\right)\right)^{\frac{\alpha}{1-\alpha}} \sigma^{\frac{\sigma}{(1-\sigma)(1-\alpha)}} \delta^{-\frac{\alpha}{1-\alpha}}$$

Take derivatives with respect to $\bar{A}$:

$$\frac{\partial y_{ss}}{\partial \bar{A}} = \frac{1}{\left(1-\sigma\right)\left(1-\alpha\right)} \frac{y_{ss}}{\bar{A}}$$

Thus:

$$\varepsilon = \frac{\bar{A}}{y_{ss}} \frac{\partial y_{ss}}{\partial \bar{A}} = \frac{1}{\left(1-\sigma\right)\left(1-\alpha\right)}$$

5 – What, in words, is the interpretation of ε? (One sentence max.) Is ε large when α is large or small? Is ε large when σ is large or small?

ε represents how much the steady-state output per capita changes in response to a change in technology. As σ or α increases, ε increases as well. The intuition for this result is spelled out, in great detail, in "Intermediate Goods and Weak Links in the Theory of Economic Development," which is a 2011 paper by Chad Jones.

PART III *(24 points)*
(3 points for each question)

A consumer, who lives for an infinite number of periods, has preferences over two types of goods. The first type of goods (called cash goods) must be purchased with cash, while the second type of goods (called credit goods) can be bought without holding money. The representative agent's preferences are given by:

$$U(c_0, d_0, c_1, d_1 ...) = \sum_{t=0}^{\infty} \beta^t \left[\alpha \log c_t + (1-\alpha) \log d_t\right] \tag{7}$$

In each period, y_t units of consumption are produced and can be divided among the two goods:

$$y_t = c_t + d_t \tag{8}$$

In Equations (7) and (8), c_t is the amount of the cash good that the consumer purchases in period t, while d_t is the amount of the credit good that he purchases in period t. Assume that y_t is an endowment that grows at rate

g per period:

$$y_t = y_0 \cdot (1+g)^t \tag{9}$$

The consumer enters period t with m_t dollars in cash, $b_t(1+R_t)$ dollars in bonds left over from the previous period (i.e., b_t are the bonds purchased in period $t-1$ that mature in period t). Assume also, that the consumer receives τ_t dollars, as a transfer from the government. In the same period, it is endowed with $P_t y_t$ dollars of the consumption good.

In period t, the consumer pays $P_t(c_t + d_t)$ to buy the cash good and the credit good. It pays b_{t+1} dollars to buy bonds for the next period. Finally, it keeps m_{t+1} dollars in cash.

Putting the last two paragraphs together, the budget constraint of the consumer is:

$$m_t + b_t(1+R_t) + P_t y_t + \tau_t = P_t(c_t + d_t) + m_{t+1} + b_{t+1} \tag{10}$$

In Equation (10), R_t is the nominal interest rate for bonds that mature in period t.

The cash-in-advance constraint states that the consumer must have enough money to purchase the cash good:

$$P_t c_t = m_t \tag{11}$$

1 – Write the Lagrangian for the constrained maximization problem of the consumer. Use λ_t for the Lagrange multiplier on the period-t budget constraint. Use μ_t for the Lagrange multiplier on the period-t cash-in-advance constraint.

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \left[\alpha \log c_t + (1-\alpha)\log d_t\right] \\
&+\lambda_t \left[b_t(1+R_t) + m_t + \tau_t + p_t y_t - p_t d_t - p_t c_t - b_{t+1} - m_{t+1}\right] \\
&+\mu_t \left[m_t - p_t c_t\right]
\end{aligned}$$

Alternatively, the Lagrange multipliers λ_t and μ_t may also multiplied by β^t. It's fine to write your Lagrangian this way, also. None of the resulting c_t's or d_t's should change by doing so.

2 – Compute the first order conditions with respect to c_t, d_t, b_{t+1}, and m_{t+1}.

To see how to take first order conditions with respect to b_{t+1} and m_{t+1}, it may be helpful to write out a couple terms of this sum:

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t \left[\alpha \log c_t + (1-\alpha)\log d_t\right] + \mu_t \left[m_t - p_t c_t\right] \\
&+\lambda_t \left[b_t(1+R_t) + m_t + \tau_t + p_t y_t - p_t d_t - p_t c_t - b_{t+1} - m_{t+1}\right] \\
= \quad &... + \beta^t \left[\alpha \log c_t + (1-\alpha)\log d_t\right] + \mu_t \left[m_t - p_t c_t\right] \\
&+\lambda_t \left[b_t(1+R_t) + m_t + \tau_t + p_t y_t - p_t d_t - p_t c_t - b_{t+1} - m_{t+1}\right] \\
&+\beta^{t+1} \left[\alpha \log c_{t+1} + (1-\alpha)\log d_{t+1}\right] + \mu_{t+1} \left[m_{t+1} - p_{t+1} c_{t+1}\right] \\
&+\lambda_{t+1} \left[b_{t+1}(1+R_{t+1}) + m_{t+1} + \tau_{t+1} + p_{t+1} y_{t+1} - p_{t+1} d_{t+1} - p_{t+1} c_{t+1} - b_{t+2} - m_{t+2}\right] \\
&+...
\end{aligned}$$

Notice where m_{t+1} and b_{t+1} appear: b_{t+1} appears in the term with the Lagrange multiplier λ_t, in addition to the term with the Lagrange multiplier λ_{t+1}. Similarly, m_{t+1} appears in the term with the Lagrange multiplier λ_t, in addition to the terms with the Lagrange multiplier λ_{t+1} and μ_{t+1}.
The first order conditions are:

$$\begin{aligned}
[c_t] : \beta^t \frac{\alpha}{c_t} &= \beta^t \mu_t P_t + \beta^t \lambda_t P_t \\
[d_t] : \beta^t \frac{1-\alpha}{d_t} &= \beta^t \lambda_t P_t \\
[m_t] : \mu_{t+1} + \lambda_{t+1} &= \lambda_t \\
[b_t] : \lambda_{t+1}(1+R_{t+1} \quad) &= \lambda_t
\end{aligned}$$

3 – Using Equation (8) plus the four first order conditions, solve for c_t and d_t in terms of y_t, $(1+R_t)$, and α.

Combine the $[m_{t+1}]$ and $[b_{t+1}]$ FOCs: $(1+R_{t+1})\lambda_{t+1} = \mu_{t+1} + \lambda_{t+1}$. If this equation holds for an arbitrary $t+1$, then it also holds for an arbitrary t: $(1+R_t)\lambda_t = \mu_t + \lambda_t$.Thus:

$$1 + R_t = \frac{\mu_t + \lambda_t}{\lambda_t}$$

Combine the $[c_t]$ and $[d_t]$ FOCs: $\frac{c_t}{d_t}\frac{(1-\alpha)}{\alpha} = \frac{\lambda_t}{\mu_t + \lambda_t} = \frac{1}{1+R_t}$. Using the market clearing condition:

$$\begin{aligned}
\frac{c_t}{y_t - c_t}\frac{(1-\alpha)}{\alpha} &= \frac{1}{1+R_t} \\
c_t(1-\alpha)(1+R_t) &= \alpha(y_t - c_t) \\
c_t(1-\alpha)(1+R_t) + \alpha c_t &= a y_t \\
c_t^* &= y_t \frac{\alpha}{(1-\alpha)(1+R_t) + \alpha}
\end{aligned}$$

Again, using the market clearing condition:

$$d_t^* = y_t \frac{(1-\alpha)(1+R_t)}{(1-\alpha)(1+R_t) + \alpha}$$

4 – For the moment, assume that $R_t \equiv R \geq 0$ is the same for all periods. Using your answer to question 3, argue why c_t grows at rate g per period. Also, using your answer to question 3, argue why d_t grows at a rate g per period. Finally, solve for P_t as a function of y_t, m_t,α, and R.

Since y_t is growing at a rate g per period an α and $1 + R_t$ are assumed to be the same for each period, we may conclude that c_t is also growing at a rate g per period. To see this, explicitly, note that:

$$\frac{c^*_{t+1}}{c^*_t} = \frac{y_{t+1}\frac{\alpha}{(1-\alpha)(1+R)+\alpha}}{y_t\frac{\alpha}{(1-\alpha)(1+R)+\alpha}} = \frac{y_{t+1}}{y_t} = 1 + g$$

Plug in the answer for c_t into the cash-in-advance constraint to get that:

$$p_t = m_t \cdot \frac{(1-\alpha)(1+R_t)+\alpha}{\alpha y_t}$$

5 – Using your first order conditions, and the fact that c_t (also d_t) grows at a constant rate across periods, find an expression relating $\frac{P_{t+1}}{P_t}$, β, g, and $(1 + R)$.

Take the $[d_t]$ first order condition across two different periods:

$$\begin{aligned} \frac{\beta^t(1-\alpha)}{d_t} &= p_t\lambda_t \\ \beta^{t+1}\frac{(1-\alpha)}{d_{t+1}} &= p_{t+1}\lambda_{t+1} \end{aligned}$$

Thus: $\beta\frac{d_t}{d_{t+1}} = \frac{p_{t+1}}{p_t}\frac{\lambda_{t+1}}{\lambda_t}$. Since $\frac{d_t}{d_{t+1}} = \frac{1}{1+g}$:$\beta = \frac{p_{t+1}}{p_t}\frac{\lambda_{t+1}}{\lambda_t}(1+g)$.
Furthermore, from the $[b_{t+1}]$ first order condition, we know that $\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{(1+R_{t+1})}$.
Again, since we are assuming that R is constant across periods:

$$\begin{aligned} \beta &= \frac{p_{t+1}}{p_t}\frac{1}{1+R}(1+g) \\ 1+R &= \frac{1}{\beta}\frac{p_{t+1}}{p_t}(1+g) \end{aligned}$$

6 – Suppose $\tau_t = \gamma m_t$, so that the per capita money supply grows at a rate γ per period (γ may be different from g). Compute the inflation rate $\pi_t = \frac{P_{t+1}}{P_t} - 1$. (Hint: Use the cash-in-advance constraint—$P_t c_t = m_t$— for periods t and $t + 1$.)

Take the cash-in-advance constraint across two consecutive periods:

$$\frac{p_{t+1}}{p_t}\frac{c_{t+1}}{c_t} = \frac{m_{t+1}}{m_t}$$

We know that $\frac{c_{t+1}}{c_t} = 1+g$ and that $\frac{m_{t+1}}{m_t} = 1+\gamma$. Thus: $\frac{p_{t+1}}{p_t} = \frac{1+\gamma}{1+g}$ and $\pi_t = \frac{1+\gamma}{1+g} - 1$. Inflation is positive if and only if $\gamma > g$ (the growth rate of the money supply is greater than the growth rate of the endowment.)

7 – Using your answers to earlier questions of the problem, solve for c_t in terms of α, β, γ, and y_t. Then solve for d_t as a function of α, β, γ, and y_t.

$$\begin{aligned}
c_t &= y_t \frac{\alpha}{(1-\alpha)(1+R)+\alpha} \\
&= y_t \frac{\alpha}{(1-\alpha)\frac{1}{\beta}\frac{p_{t+1}}{p_t}(1+g)+\alpha} \\
&= y_t \frac{\alpha\beta}{(1-\alpha)(1+\gamma)+\alpha\beta} \\
d_t &= y_t - y_t \frac{\alpha\beta}{(1-\alpha)(1+\gamma)+\alpha\beta} \\
&= y_t \frac{(1-\alpha)(1+\gamma)}{(1-\alpha)(1+\gamma)+\alpha\beta}
\end{aligned}$$

8 – What is $\frac{\partial c_t}{\partial \gamma}$? What is $\frac{\partial d_t}{\partial \gamma}$? What is the interpretation of these results? (2 sentences max)

$$\frac{\partial c_t}{\partial \gamma} = -y_t \frac{\alpha\beta(1-\alpha)}{\left((1-\alpha)(1+\gamma)+\alpha\beta\right)^2} < 0$$
$$\frac{\partial d_t}{\partial \gamma} = y_t \frac{\alpha\beta(1-\alpha)}{\left((1-\alpha)(1+\gamma)+\alpha\beta\right)^2} > 0$$

Money growth causes the consumer to purchase less of the cash good and more of the credit good; the cash good requires money and holding money is costly; the cost of holding money increases as γ increases.

PART IV *50 points*
(*10 points for each question*)

For each of those statements, say whether it is true, false, or uncertain (explain why and expose the necessary background).

1 – Children are like an inferior good.

TRUE

a – Definition of an inferior good: it is a good whose demand decreases with income.

b – When one looks at the data, the industrial revolution has come with first an increase in income per capita, followed by a decrease in mortality and in fertility. We observe the same phenomenon in developing countries. This is called the *demographic transition*.

c – In the model of the course, we have been able to generate a decrease in fertility even if children were not inferior goods. Because utility was quasi-linear, the demand for chidden had zero income elasticity. As the price of children was the price of food, it is the increase in the relative price of food during the industrial revolution that is in the model responsible for the gradual decrease in fertility.

d – It is often argued that one should decompose children into quality and quantity components. Quality refers to the level of education. At the same time that the quantity of children has fallen, the quality has increased, such that people are overall spending as much as before on children (perhaps more). In that sense, only the quantity of children is an inferior good.

2 – There was no growth on income during the Malthusian regime.

TRUE

a – The Malthusian regime is defined as the economic phase that followed the hunter-gatherers societies and the invention of agriculture and preceded the industrial revolution.

b – That regime is characterized by growth in total income and population, but stagnation of income per capita.

c – In the Malthusian regime, comparative statics is as follows: technical progress increases population, not income per capita; death rate increases income per capita (example of the Black Death in England, 1348), fertility rate decreases income per capita.

d – A model that can represent that regime is given by the three equations: (1) the birth rate b is high when income per capita y is high: $b = b(y),\ b' > 0$; (2) the mortality rate m is low when income per capita y is high: $m = m(y),\ m' < 0$; (3) population P is low when income per capita y is high: $P = P(y),\ P' < 0$.

e – Malthus was Professor of Economy. He wrote in 1798 *An Essay on the Principle of Population*. He argued that population growth generally expanded in times and in regions of plenty until the size of the population relative to the primary resources caused distress. Two types of checks hold population within resource limits: positive checks, which raise the death rate (*hunger, disease and war*) and preventive checks, which lower the birth rate (*abortion, birth control, prostitution, postponement of marriage and celibacy*).

f – A good analogy is with a population of rabbits on an island. The rabbits are always at their subsistence level. If it rains, there is more grass, more young rabbits survive and population increases. If it is dry, young rabbits do not survive and population decreases. In both cases, grass per head is constant.

3 – When the probability at birth for life length L to be larger than ℓ is $P(L > \ell) = e^{-m\ell}$, then life expectancy at age 20 is equal to life expectancy at birth *minus* 20.

FALSE: L is distributed with cumulative function $F(\ell) = 1 - P(L > \ell) = 1 - e^{-m\ell}$, which means that the probability at birth that life length is shorter than ℓ is $1 - e^{-m\ell}$. The density of the random variable "life length" is $f(\ell) = F'(\ell) = me^{-m\ell}$. The probability at a that life is longer that $\ell > a$ is

$$P(L > \ell \mid L > a) = \frac{P(L > \ell \text{ and } L > a}{P(L > a)} = \frac{e^{-m\ell}}{e^{-ma}} = e^{-m(\ell - a)} = P(L > \ell - a)$$

and density of life length as of age a is the derivative of $1 - e^{-m(\ell-a)}$: $f_a(\ell) = me^{-m(\ell-a)} = e^{ma} f(\ell)$.
Let $\mathcal{E}_a$ be life expectancy at age a. $\mathcal{E}_a$ is given by $\mathcal{E}_a = \int_a^\infty \ell f_a(\ell) d\ell = e^{ma} \int_a^\infty \ell f(\ell) d\ell = e^{ma} \int_a^\infty \ell m e^{-m\ell} d\ell$. We solve this integral integrating by parts. Let $u(\ell) = \ell$ and $v'(\ell) = me^{-m\ell}$.
We have $u'(\ell) = 1$ and $v(\ell) = -e^{-m\ell}$.
With these notations,

$$\begin{aligned}
\mathcal{E}_a &= e^{ma} \int_a^\infty uv' d\ell = e^{ma} \Big[uv \Big]_a^\infty - e^{ma} \int_a^\infty u'v d\ell = e^{ma} \Big[-\ell e^{-m\ell} \Big]_a^\infty - e^{ma} \int_a^\infty -e^{-m\ell} d\ell \\
&= ae^{-ma} e^{ma} + e^{ma} \Big[-\frac{1}{m} e^{-m\ell} \Big]_a^\infty = a + \frac{1}{m} = \mathcal{E}_0 + a
\end{aligned}$$

Therefore, life expectancy at age 20 is equal to life expectancy at birth *plus* 20.
– *Acceptable answer (5 points* : When the life length distribution is exponential, we have a property of memorylesness: the probability of dying per period is independent of age, and takes the value m. In that case, life expectancy at age 20 is equal to life expectancy at birth *plus* 20.

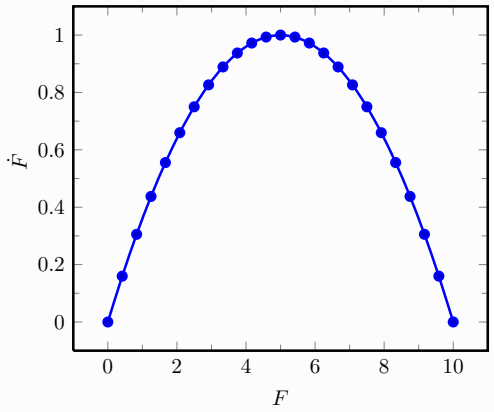
4 – Assume that the law of motion of a resource F (say a forest) is given by $\dot{F}(t) = rF(t)\left(1 - \frac{F(t)}{K}\right)$. In this case, there are only two steady state level of F.

TRUE

a – A steady state correspond to a forest size F such that $\dot{F}(t) = 0$. Solving that equation gives $F = 0$ or $F = K$. There are two steady states that correspond to maximum size of the forest or complete deforestation.

b – The law of motion of the forest is give by a logistic function. K is the "carrying capacity", *i.e.* maximum size of the resource, r is the regeneration rate of the forest.

c – Graphically, it can be represented as shown below



5 – Having a high growth of money supply is a way to guarantee higher growth of real income in the long run.

UNCERTAIN

a – We gave the following result in class : In the long run, there is no correlation between the growth rates of money and real output. This holds across all definitions of money, but not for a subsample of OECD countries, where the correlation seems to be positive.

b – To think of the long run link between money creation and growth, we can use the equation of exchange $Mv = PY$ which gives

$$\underbrace{\frac{\dot{M}}{M}}_{\text{Money growth}} + \underbrace{\frac{\dot{v}}{v}}_{\text{velocity growth}} = \underbrace{\frac{\dot{P}}{P}}_{\text{Inflation}} + \underbrace{\frac{\dot{Y}}{Y}}_{\text{Real output growth}}$$

c – Data show that *in the long run*, $\frac{\dot{Y}}{Y}$ is essentially disconnected from $\frac{\dot{M}}{M}$ and $\frac{\dot{P}}{P}$, and that $\frac{\dot{P}}{P}$ moves essentially one to one with $\frac{\dot{M}}{M}$.

d – Hyperinflation are episodes of high money creation that generally correspond to economic low growth (or recession). Il is not clear that the causality in unidirectional: on the one side, recession implies low revenues for the government, and therefore high money creation; on the other side, high money creation implies hyperinflation, which disrupts the economy.