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# Exam

*Instructions:*

- *This is a 1h00 hours exam.*
- ***BE SURE TO PUT THE STICKER ON THE EXAM.***
- *There are a total of 100 points on the test.*
- *You have to answer all the questions.*
- ***Use ONLY the space provided for answering the questions.***
- ***You must write legibly.***

## Part I: Skilled and unskilled labor *(70 points)*
*(7 points for each question)*

Consider an economy with two types of households. There are $N_S = 100$ skilled households and $N_U = 200$ unskilled households. Each household owns a single, distinct firm with the following technology

$$y = k^{\frac{1}{3}} \left(l_d^U\right)^{\frac{1}{3}} \left(l_d^S\right)^{\frac{1}{3}} \tag{1}$$

In Equation (1), $k$ is the (exogenously given) amount of capital that each household owns. In the equation, $l_d^U$ refers to the amount of unskilled labor that the firm demands, and $l_d^S$ refers to the amount of skilled labor that the firm demands. The total amount of the consumption good produced, in the economy, equals the sum of the outputs of the 300 firms.

Let $w^S$ denote the wage paid to skilled workers, and $w^U$ the wage of unskilled workers, in the labor market. Note that these two wages may be different from one another. The profit function for each firm is:

$$\pi = py - w^S l_d^S - w^U l_d^U \tag{2}$$

In Equation (2), $p$ represents the price of the consumption good.

Finally, households of both types have a common utility function over consumption and the fraction of their time supplied to the labor market:

$$u^S(c^S, l_s^S) = \log\left(c^S\right) + \log\left(1 - l_s^S\right) \tag{3}$$
$$u^U(c^U, l_s^U) = \log\left(c^U\right) + \log\left(1 - l_s^U\right) \tag{4}$$

Equation (3) gives the utility function of the skilled household, and Equation (4) gives the utility function of the unskilled household.

**1** – For this problem set $p = 1$. Why can we do this? With $p$ set to 1, what are the units of $w^U$ and $w^S$?

**2** – Define a competitive equilibrium in this economy. Be precise.

**3** – Determine the optimal labor demand of skilled and unskilled workers for each firm, each as a function of $w^U$, $w^S$, and $k$.

**4 –** Plug the $l_d$'s back into the profit function to get the maximized profit of each firm.

**5 –** Write the Lagrangian of the unskilled household, assuming that it earns a wage $w^U$ in the labor market. Take first order conditions.

**6 –** Solve for $l_s^U$, (the amount of labor supplied per unskilled household) as well as $c^U$.

**7 –** Solve for $l_s^S$ and $c^S$. (Hint: Remember that the only difference between skilled and unskilled households is the wage they earn per unit of labor supplied.)

**8** – State the two labor-market clearing conditions (one for skilled labor and one for unskilled labor).

**9** – Use the labor-market clearing condition for unskilled labor to find one equation relating equilibrium wages $w^{U*}$ and $w^{S*}$.

**10** – Use the labor-market clearing condition for skilled labor to find a second equation relating $w^{U*}$ and $w^{S*}$. Use this equation, and your answer from part 9, to solve for $w^{U*}$ and $w^{S*}$.

## Part II: The Malthusian regime *(30 points)*
*(10 points for each question)*

**1 –** Explain what do economists mean by the Malthusan regime.

**2 –** A simple model of the Malthisan regime is given by the following four equations (where the sign of the derivatives are indicated under curly brackets (*parenthèses* in French))

**Demography (1)** $b =$ birth rate, $y =$ income per capita : $b = b \underbrace{(y)}_{+}$

**Demography (2)** $m=$ mortality rate : $m = m \underbrace{(y)}_{-}$

**Technology** $P =$ Population : $P = P \underbrace{(y)}_{-}$

**Population dynamics** : $\dot{P} = (b - m)P$

Comment those four equations.

**3 –** Use a graphical analysis to show what happens if initial output $y_0$ is above its steady star level $y^\star$. Comment.