2014-2015 – Licence 3 – Macro

Lecture 5 : (Very) Long Run Growth

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Changes from version 1.0 are in red

Disclaimer

These are the slides I am using in class. They are not self-contained, do not always constitute original material and do contain some "cut and paste" pieces from various sources that I am not always explicitly referring to (not on purpose but because it takes time). Therefore, they are not intended to be used outside of the course nor to be distributed. Thank you for signaling me typos or mistakes at Franck.Portier@TSE-fr.eu.

0. Introduction

What is this lecture about ?

- In this Lecture, we study the very long run dynamics of economies.
- First we will identify three stages in economic history :
  - the Malthusian regime
  - The Industrial Revolution (and the demographic transition)
  - The Great Divergence
- Then we will use models to understand some of those evolutions (not the great Divergence)
  - A demographic model of the Malthusian regime
  - A fully specified general equilibrium model of Unified Growth Theory

0. Introduction

How is is related to the previous ones (of Thomas) ?

- We will come back with more detail on the Malthusian regime of growth
- We will be more explicit in studying a dynamic model of population
- We will use an OLG model (OverLapping Generations) to give a fully specified general equilibrium model of Unified Growth Theory

0. Introduction

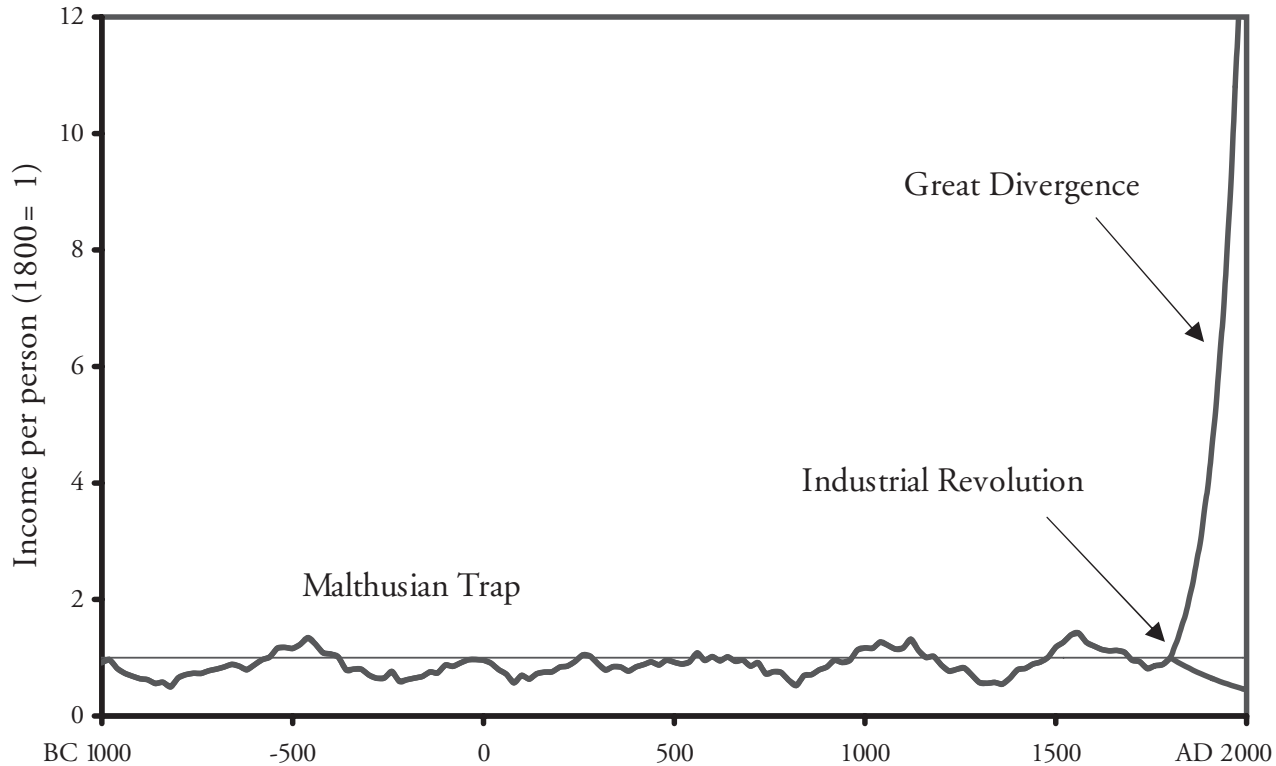
Plan of the Lecture

1. The Big Picture
2. The Malthusian Regime
3. The Industrial Revolution
4. The Great Divergence
5. Demographics and the simple functioning of the Malthusian regime
6. A (quite) simple mathematical model of life expectancy
7. A simple model of the (very) long economic history
8. Summary
9. References

1. The big picture

Economic history of the world in one graph

FIGURE 1: Income per capita from 1000 BC to 2000 AD (CLARK [2007])



Note : B.C. = Before Christ, A.D. = Anno Domine

1. The big picture

The three stages of economic history

1. Malthusian Regime (before 1760) : income per capita is roughly constant, close to subsistence level. Changes in the availability of resources translate into population changes
2. Industrial Revolution and the demographic transition (1760-1900) : rapid economic growth fueled by increasing production efficiency + increase of population smaller than the increase of total income.
3. The Great Divergence (after 1900) : some countries became unprecedentedly rich while standards of living declined or grew slowly in other.

2. The Matlhusian regime

Reverend Thomas Robert Malthus (1766-1834)

- Professor of History and Political Economy
- 1798 : *An Essay on the Principle of Population*
- He argues that population growth generally expanded in times and in regions of plenty until the size of the population relative to the primary resources caused distress.

Figure 2: Malthus



2. The Matlhusian regime

Reverend Thomas Robert Malthus (1766-1834)

- Two types of checks hold population within resource limits :
  - × positive checks, which raise the death rate : *hunger, disease and war*.
  - × preventive checks, which lower the birth rate : *abortion, birth control, prostitution, postponement of marriage and celibacy*.

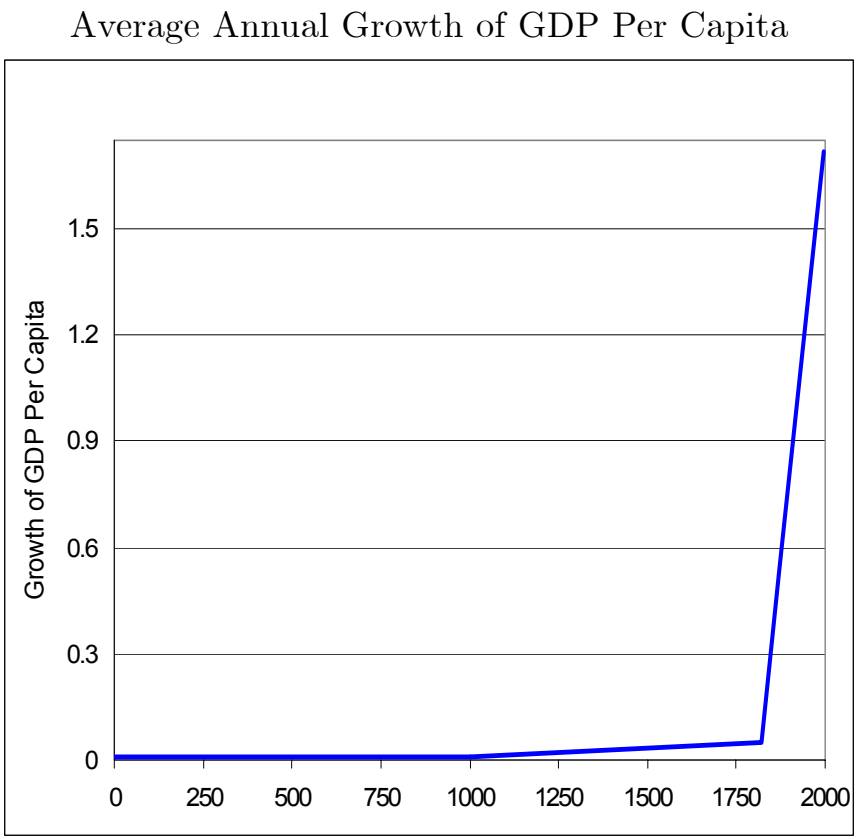
2. The Matlhusian regime

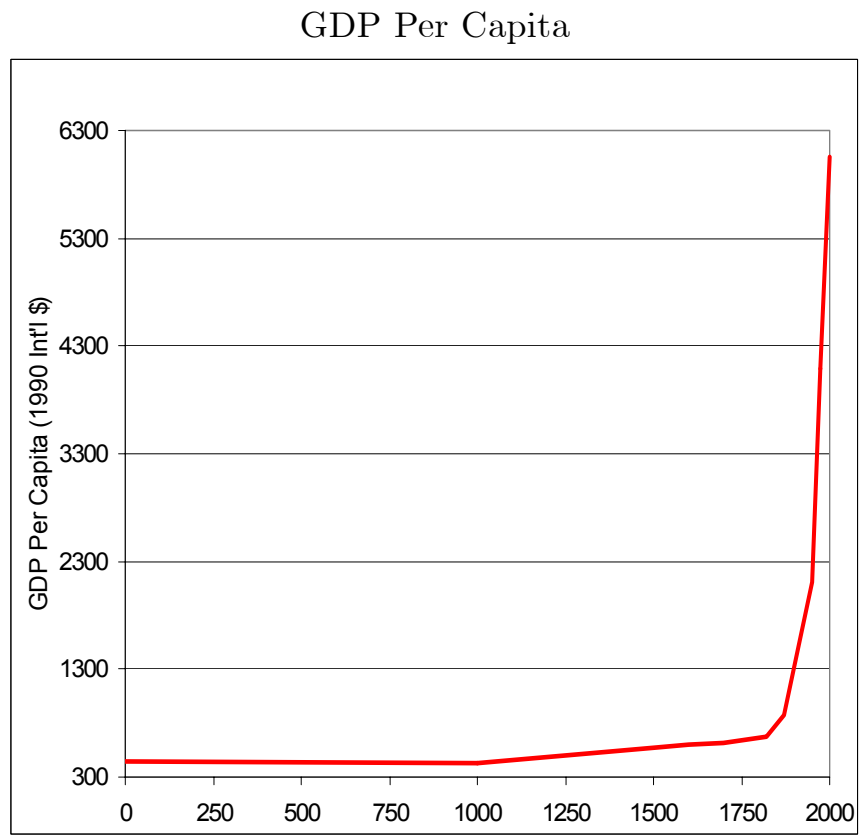
In a nutshell

- Humans struggled for life
- Technological progress was insignificant
- Technological progress and land expansion caused an increase in the size of the population, not in the standards of living.
- Income per capita stayed in the proximity of a subsistence level.
- We'll see later in this lecture a formal model of the Malthusian regime

2. The Matlhusian regime

FIGURE 3: Income per capita dynamics (GALOR [2005])



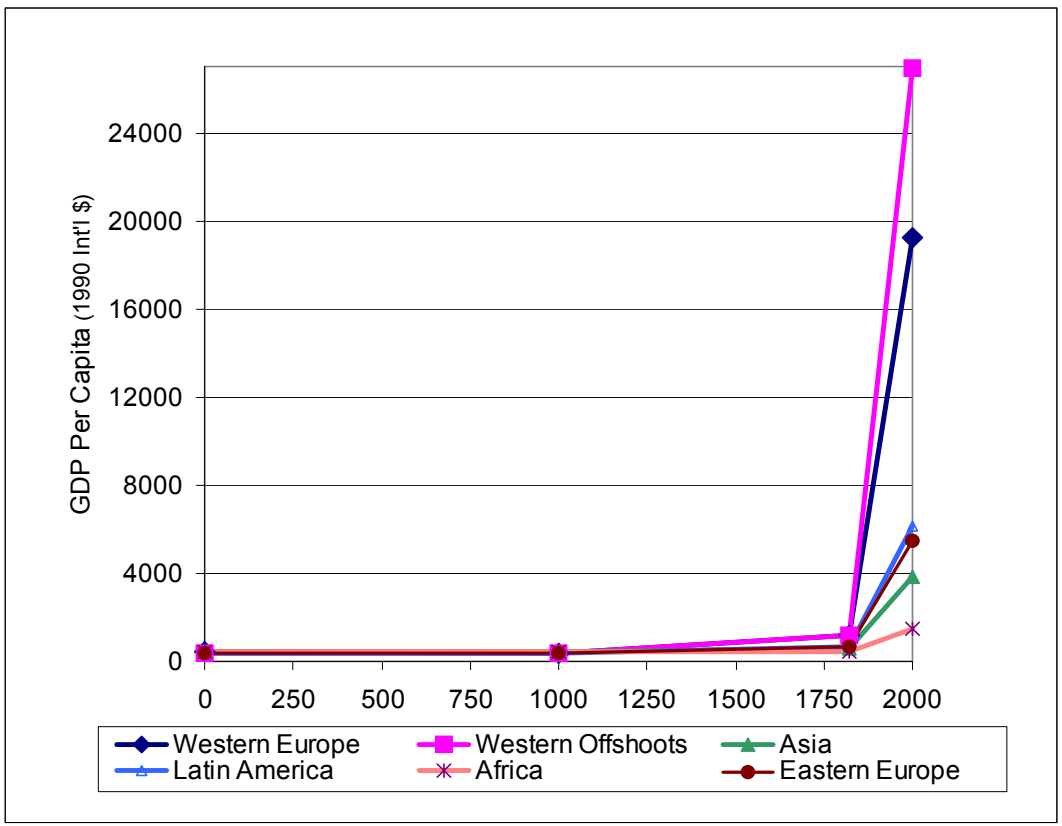


2. The Matlhusian regime

Income per capita

- Average growth rate of income per capita is negligible : 0.05% a year between 1000 and 1820 (Maddison)
- Standards of living do not vary much across countries

FIGURE 4: Across the world (GALOR [2005])

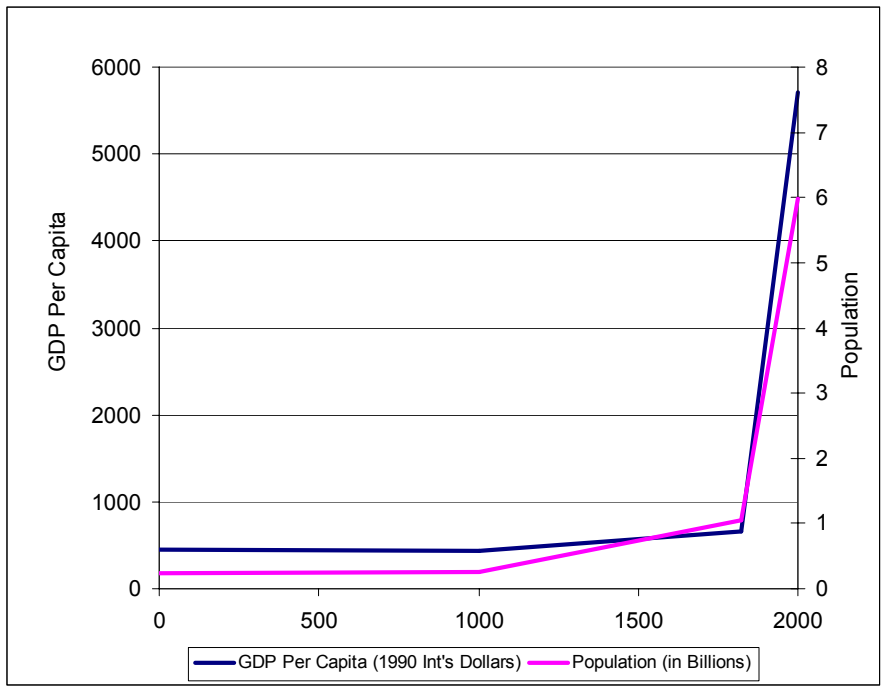


2. The Matlhusian regime

Population and income

- World population was 231 millions in 1 AD and 268 millions in 1000 AD, then 438 millions in 1500 AD
- Resource expansions after 1500 $\rightsquigarrow$ 1041 millions people in 1820

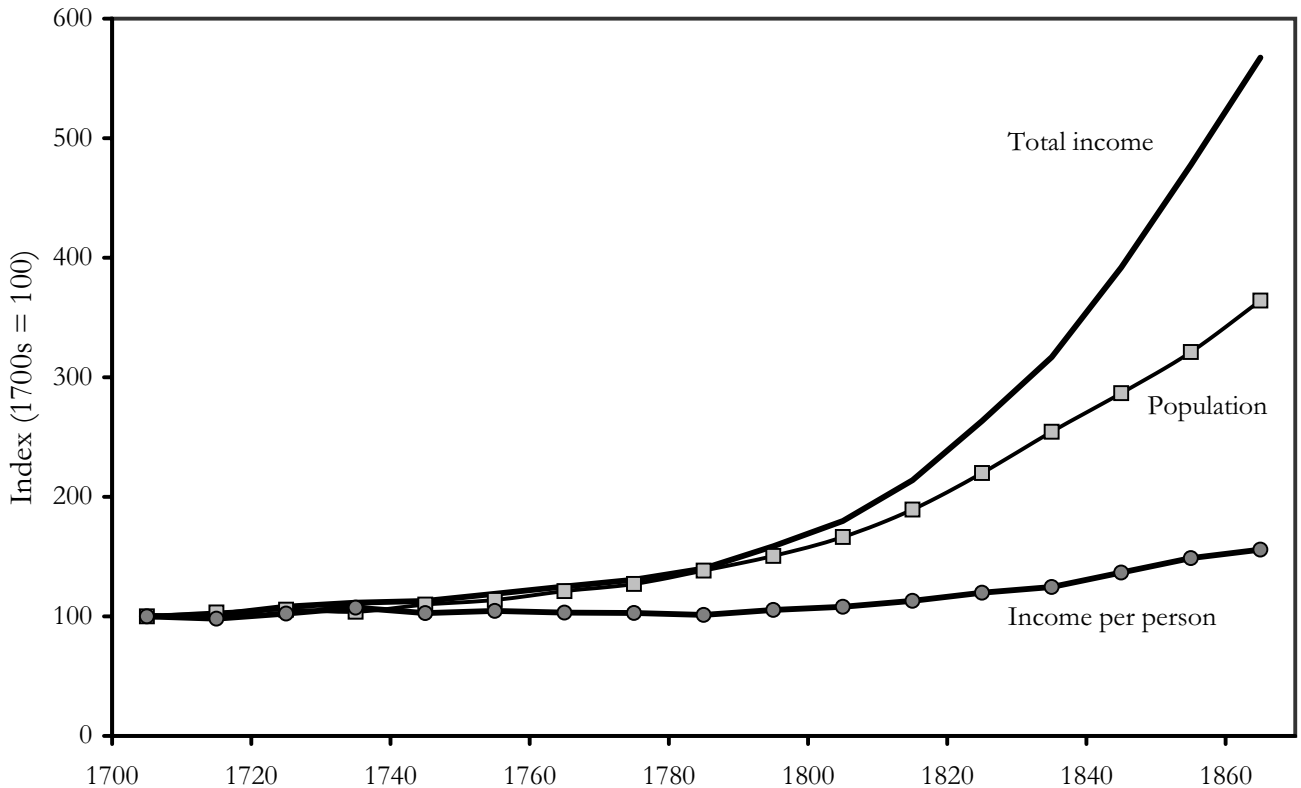
FIGURE 5: World population and income per capita from 1 AD to 2000 (GALOR [2005])



2. The Matlhusian regime

Population and income

FIGURE 6: Population and economic growth in England, 1700s-1860s (CLARK [2007])

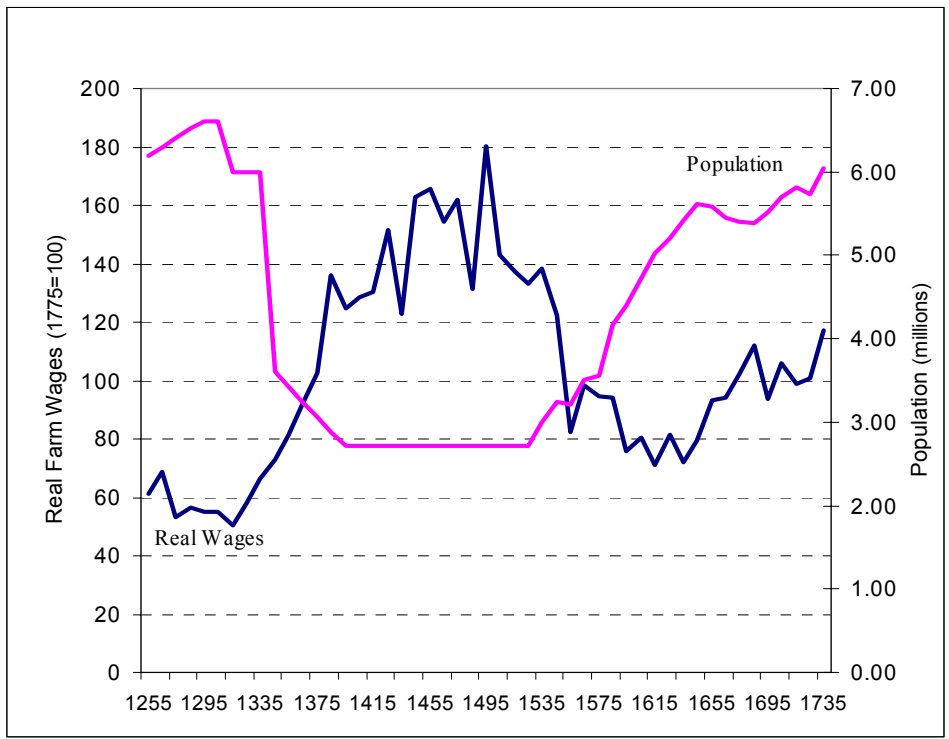


2. The Matlhusian regime

Example I : The English Black Death (1348-1349) and after

- English Black Death (1348-1349) : populations went down from 6 millions to 3.5 millions
- Increase in the land/labor ratio $\rightsquigarrow$ the real wage tripled for 150 years
- This caused an increase in population $\rightsquigarrow$ in the 1560s, the real wage is back to the pre-plague level.

FIGURE 7: Population and Real Wages in England (GALOR [2005])

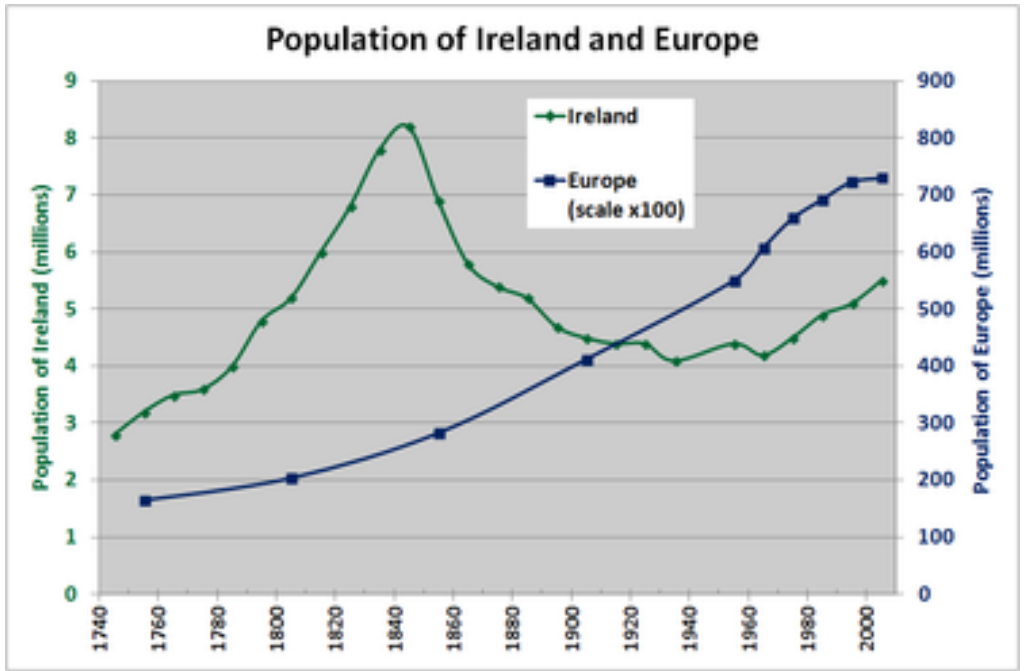


2. The Matlhusian regime

Example II : The Great Famine in Ireland (1845-1852)

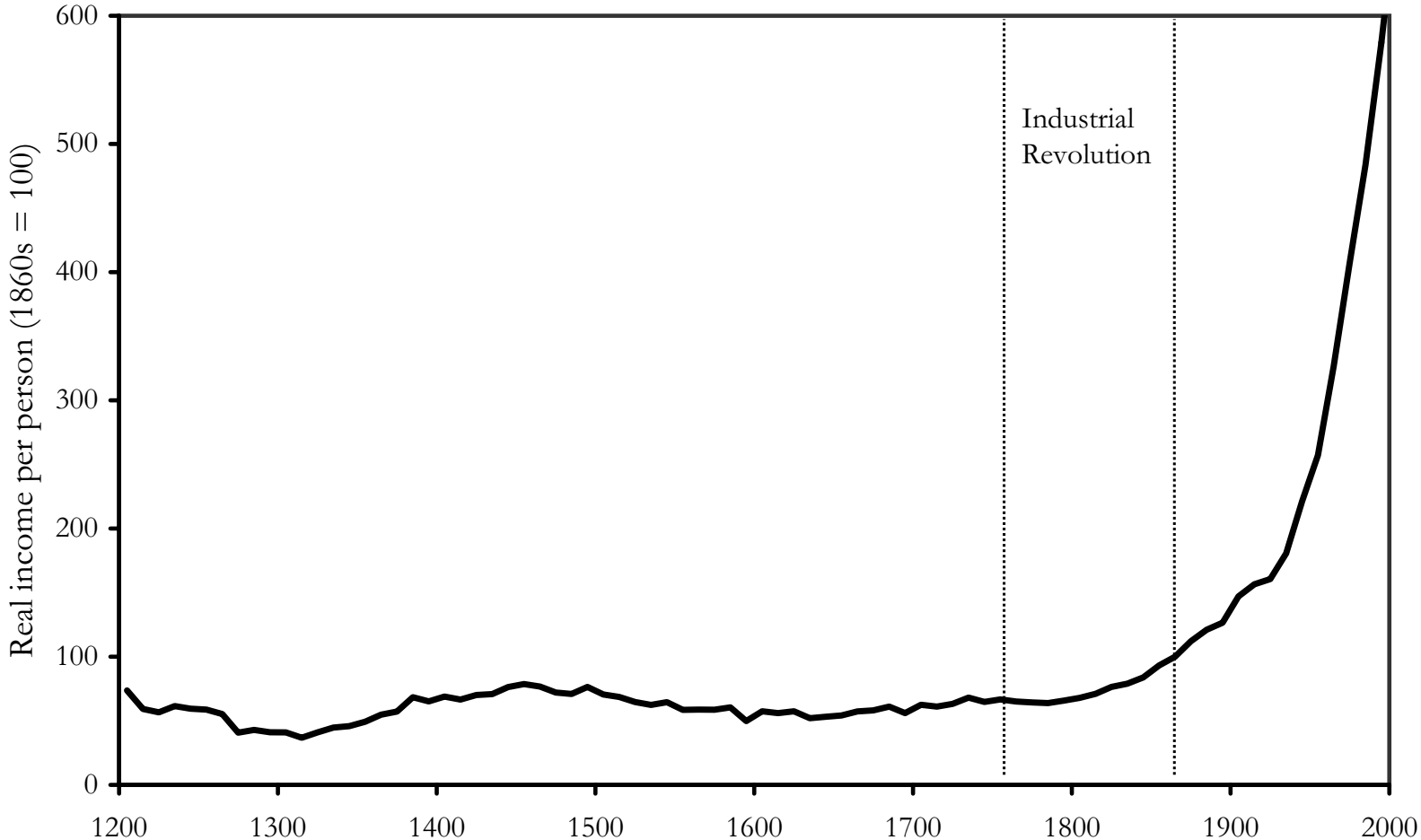
- Introduction of the potato in Ireland in the middle of the 17th century $\rightsquigarrow$ population increase
- Potato desease ("potato bligh" (*mildiou*))
- About 1 million people died + massive emigration $\rightsquigarrow$ population decline.

FIGURE 8: Evolution of population and the Great Famine (WIKIPEDIA)



3. The Industrial Revolution

FIGURE 9: Real income per person in England 1260s-2000s (CLARK [2007])



3. The Industrial Revolution

In a nutshell

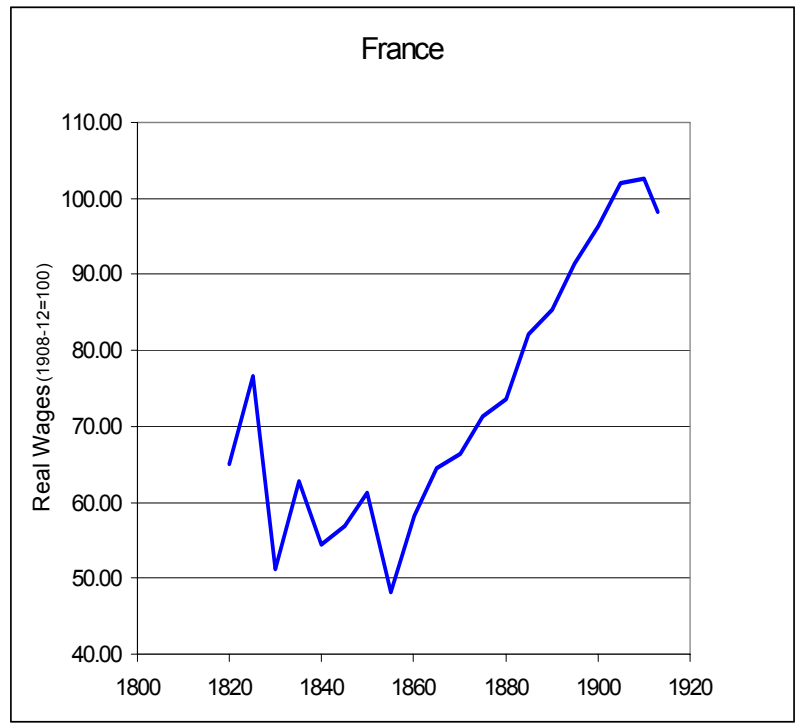
- It is a period of transition to new manufacturing processes
- Started in England around 1760 and went on up to 1840
- Spread over the continent (Belgium Wallonia first) and to the U.S.A.
- The pace of technological progress increased with industrialization
- But it is also a time of great improvement in agricultural productivity (also an *agricultural revolution*)
- Simultaneous population and income per capita growth

- Textile : Mechanized cotton spinning powered by steam or water increased the output of a worker by a factor of about 1000.
- Steam power : The efficiency of steam engines increased so that they used between one-fifth and one-tenth as much fuel.
- Iron Making : The substitution of coke for charcoal greatly lowered the fuel cost of iron.

3. The Industrial Revolution

FIGURE 10: Real Wages in England and France During the take-off from the Malthusian Epoch (GALOR [2005])





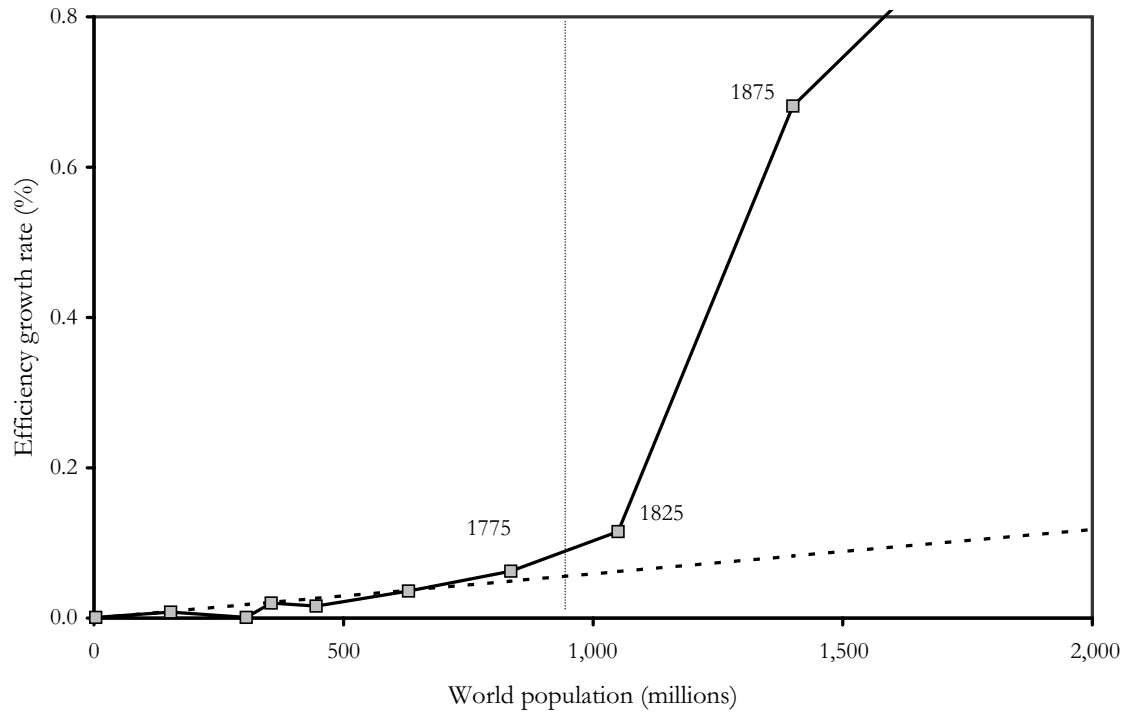
3. The Industrial Revolution

Technology and the demographic change

$$Y_t = A_t F(K_t, L_t)$$

- A measures efficiency (*Solow residual*)

FIGURE 11: World population and growth rate of efficiency (CLARK [2007])

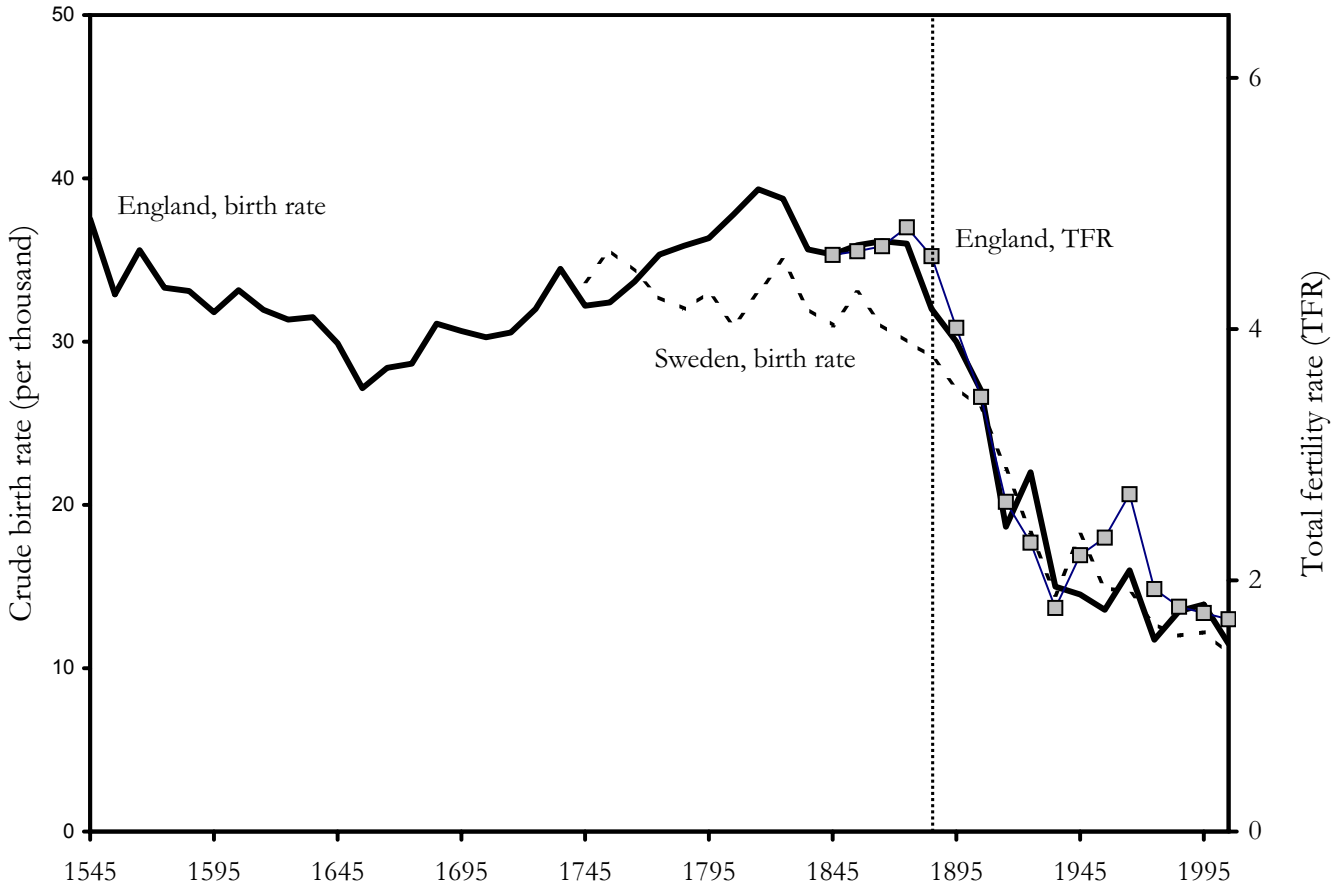


- Let's have a look at two measures of birth rates.
- Crude birth rate indicates the number of live births occurring during the year, per 1000 population estimated at midyear.
- Total fertility rate is the average number of children that would be born to a woman over her lifetime if :
  - × she were to experience the exact current age-specific fertility rates through her lifetime, and
  - × she were to survive from birth through the end of her reproductive life.

3. The Industrial Revolution

The demographic change

FIGURE 12: Demographic transition (CLARK [2007])



4. The Great Divergence

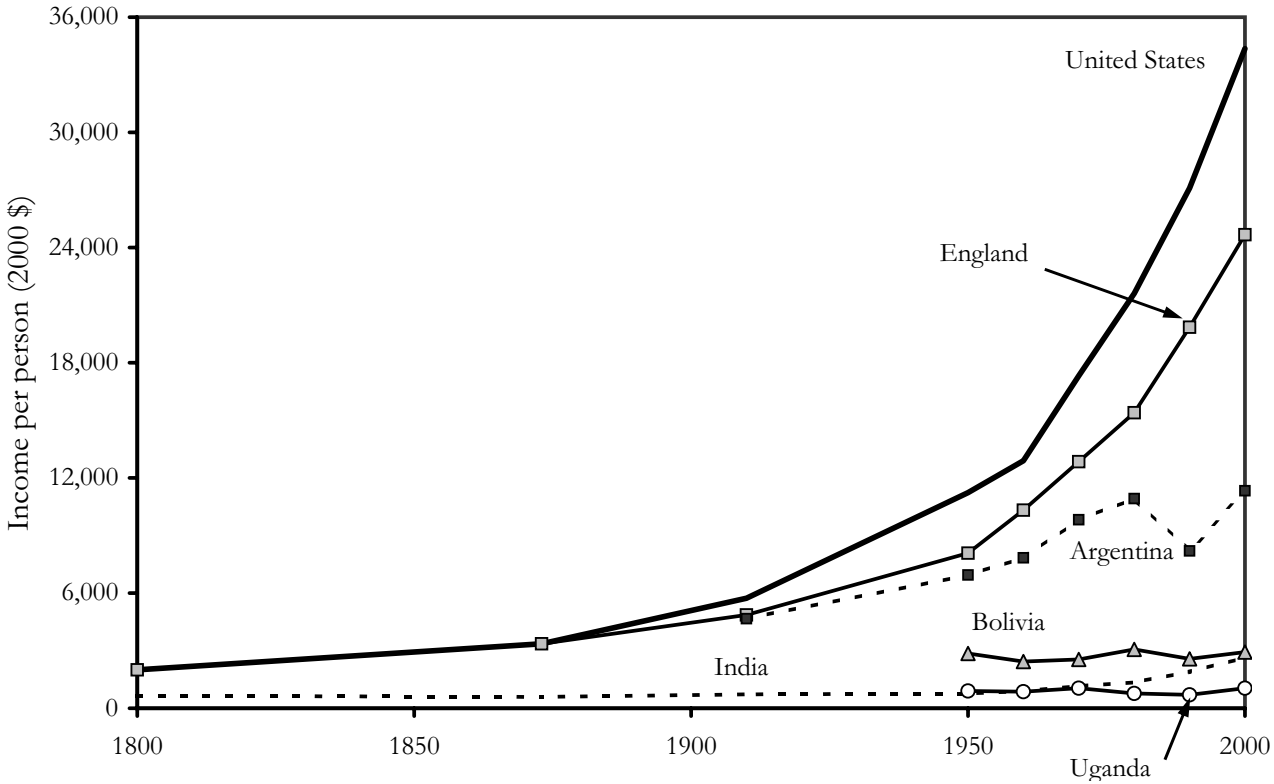
- Last two centuries : dramatic changes in the distribution of income and population across the globe.
- Inequality in the world economy was negligible till the 19th century.

TABLE 1: Ratio of GDP per capita between the richest region and the poorest region in the world (GALOR [2005])

Year	1000	1500	1820	1870	1913	1950	2001
Ratio	1.1 :1	2 :1	3 :1	5 :1	9 :1	15 :1	18 :1

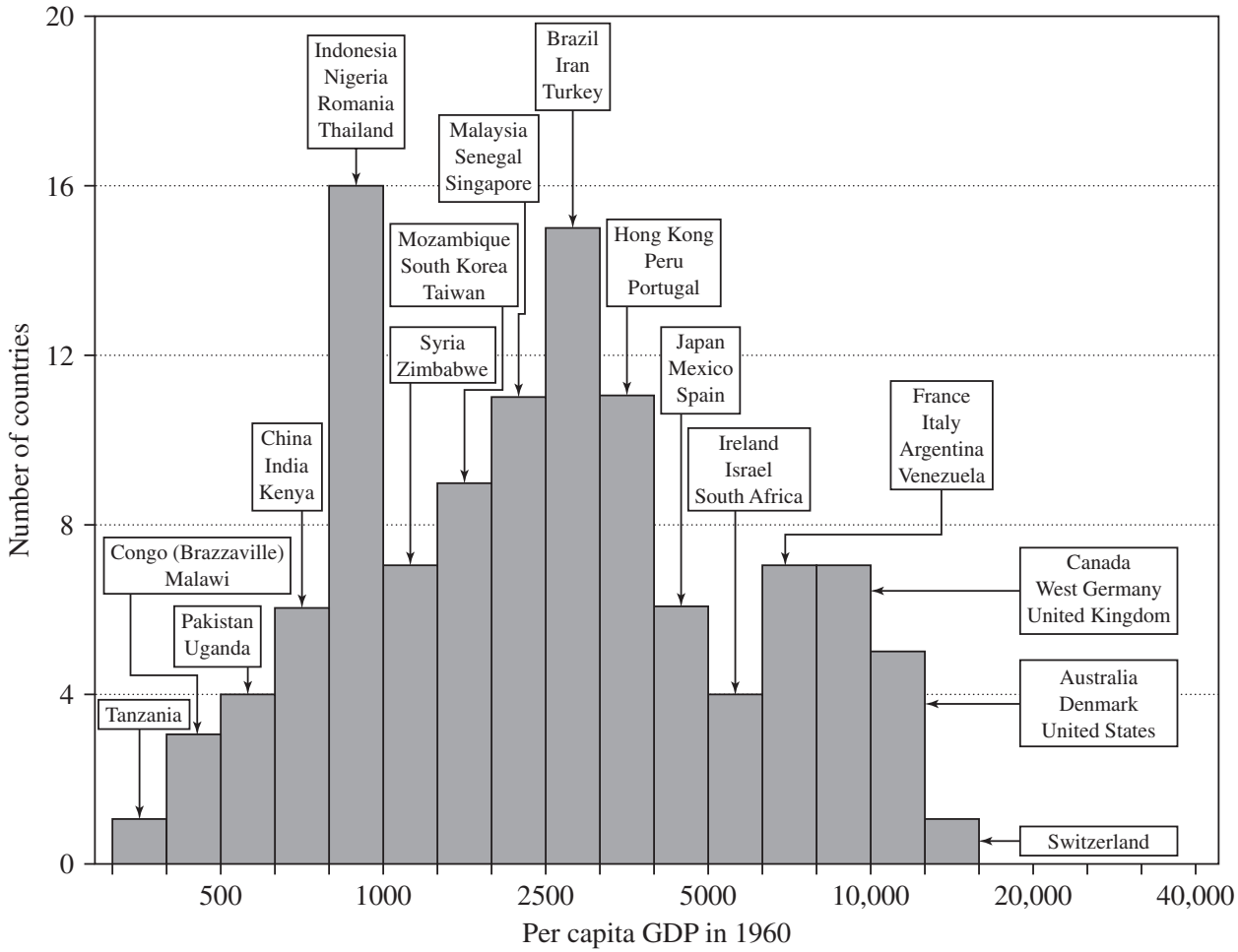
4. The Great Divergence

FIGURE 13: Some countries took-off, some did not (CLARK [2007])



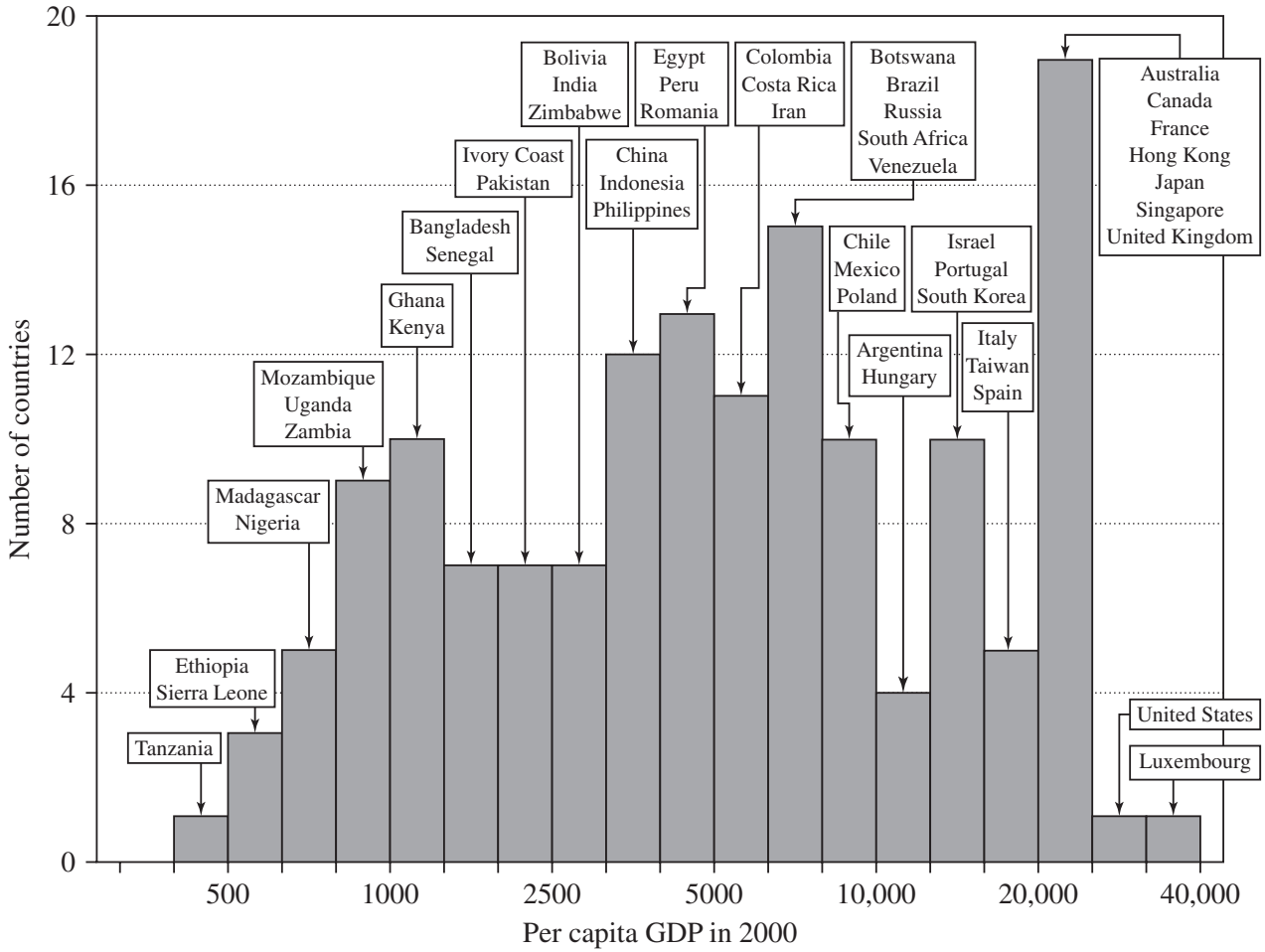
4. The Great Divergence

FIGURE 14: Inequalities across countries in 1960 (BARRO & SALA I MARTIN [2004])



4. The Great Divergence

FIGURE 15: Inequalities across countries in 2000 (BARRO & SALA I MARTIN [2004])



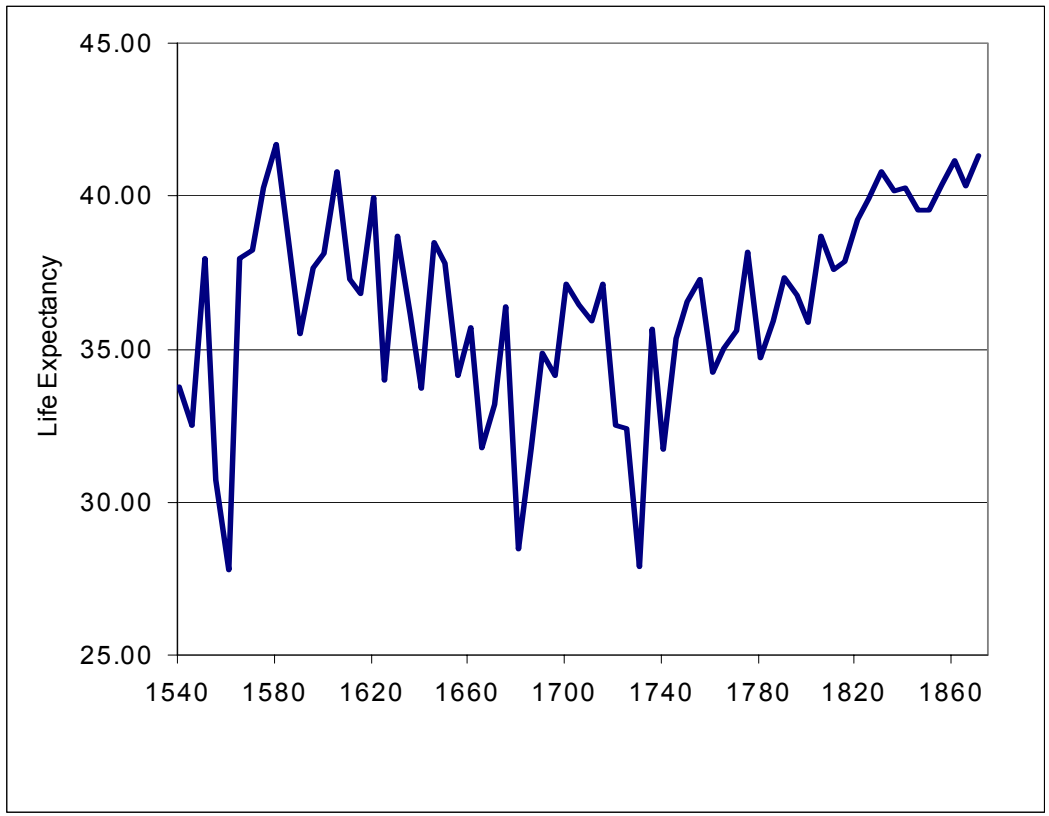
4. The Great Divergence

- Those facts raise a lot of questions :
  - × Why poor countries are not converging to the income per capita of the rich ?
  - × If $Y = AK^{\alpha}L^{1-\alpha}$, why doesn't capital fly from *capital-abundant-low-marginal-productivity-of-capital* to *capital-poor-high-marginal-productivity-of-capital* ones ?
  - × Marginal productivity of capital $= A\left(\frac{L}{K}\right)^{1-\alpha}$
  - × Are there non economic factors behind the non convergence (institutions ? geography ?)
- A lot of fascinating questions that we will not address in this lecture
- This is the field of *Growth Theory* and *Development Economics*

5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

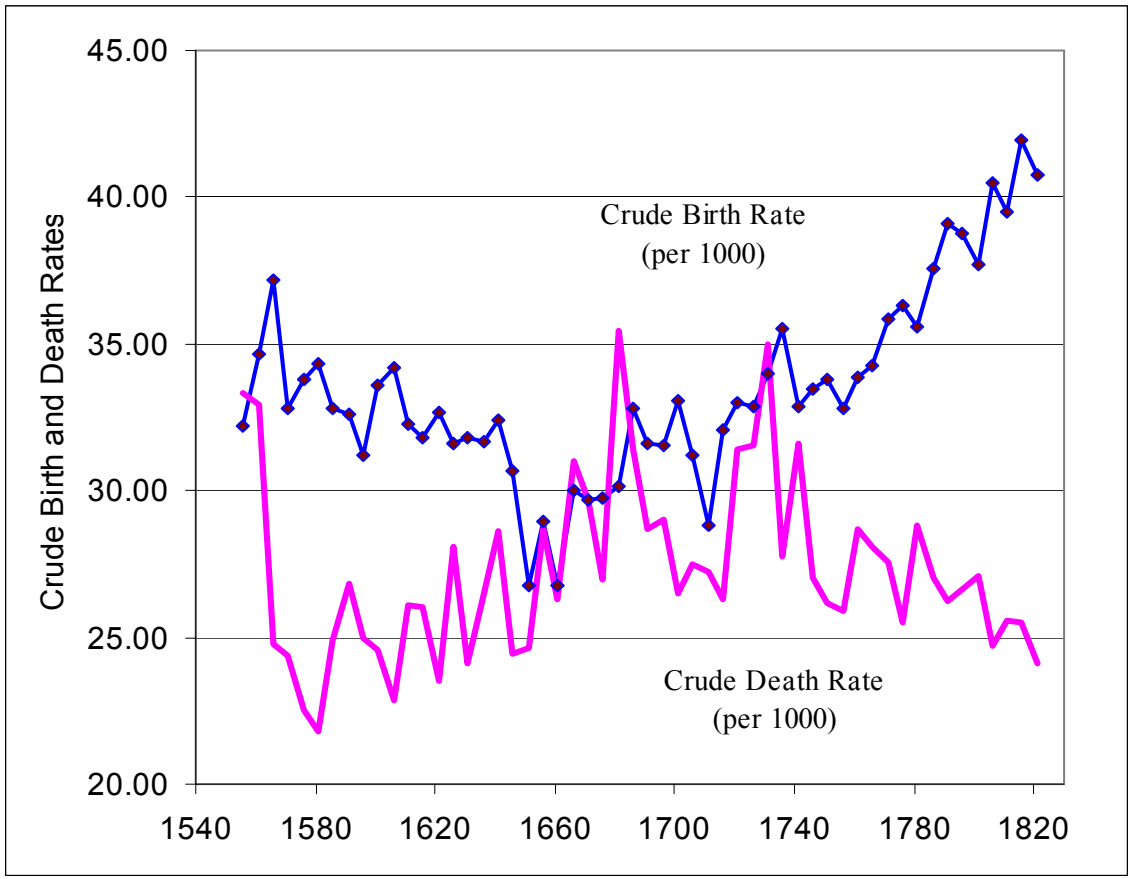
FIGURE 16: Life expectancy (at birth) in England (GALOR [2005])



5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

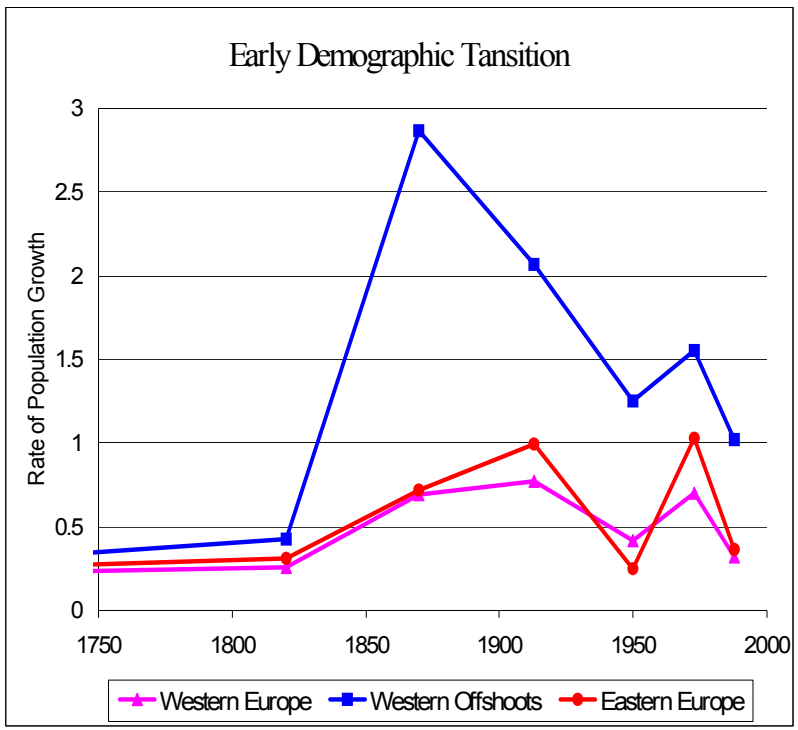
FIGURE 17: Crude birth and death rates in England (GALOR [2005])

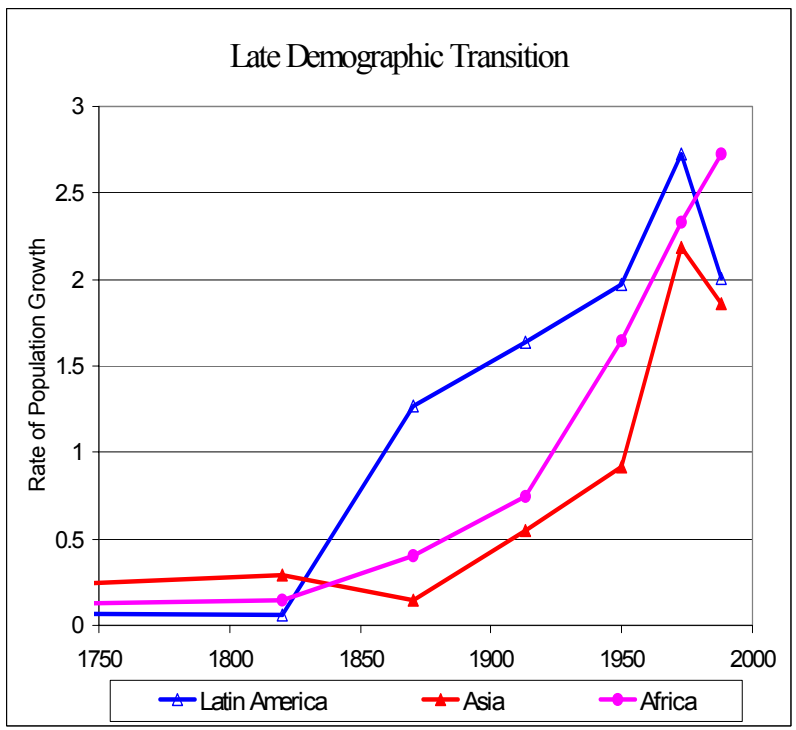


5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

FIGURE 18: Population growth (GALOR [2005])

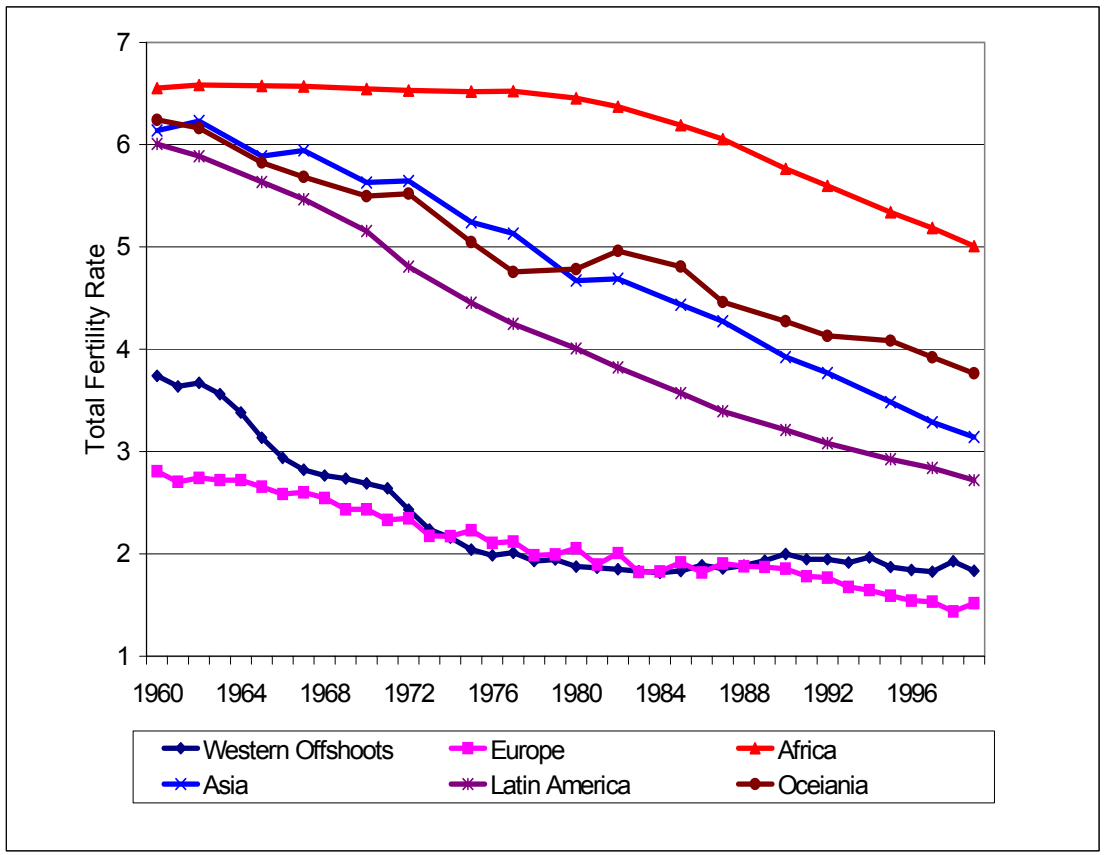




5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

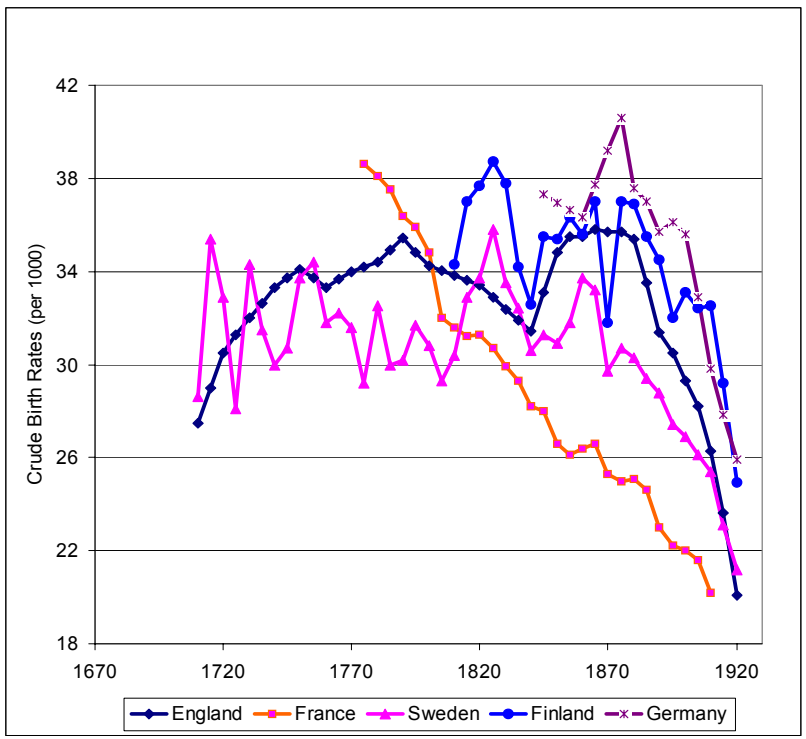
FIGURE 19: Total fertility rates (GALOR [2005])

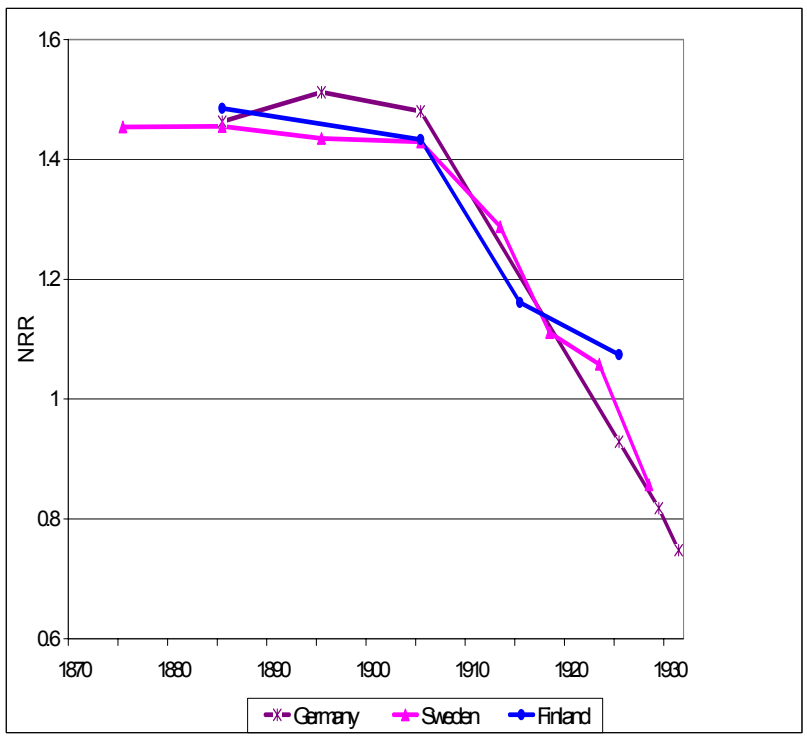


5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

FIGURE 20: Crude birth rate and Net Reproduction Rate (NRR) (GALOR [2005])



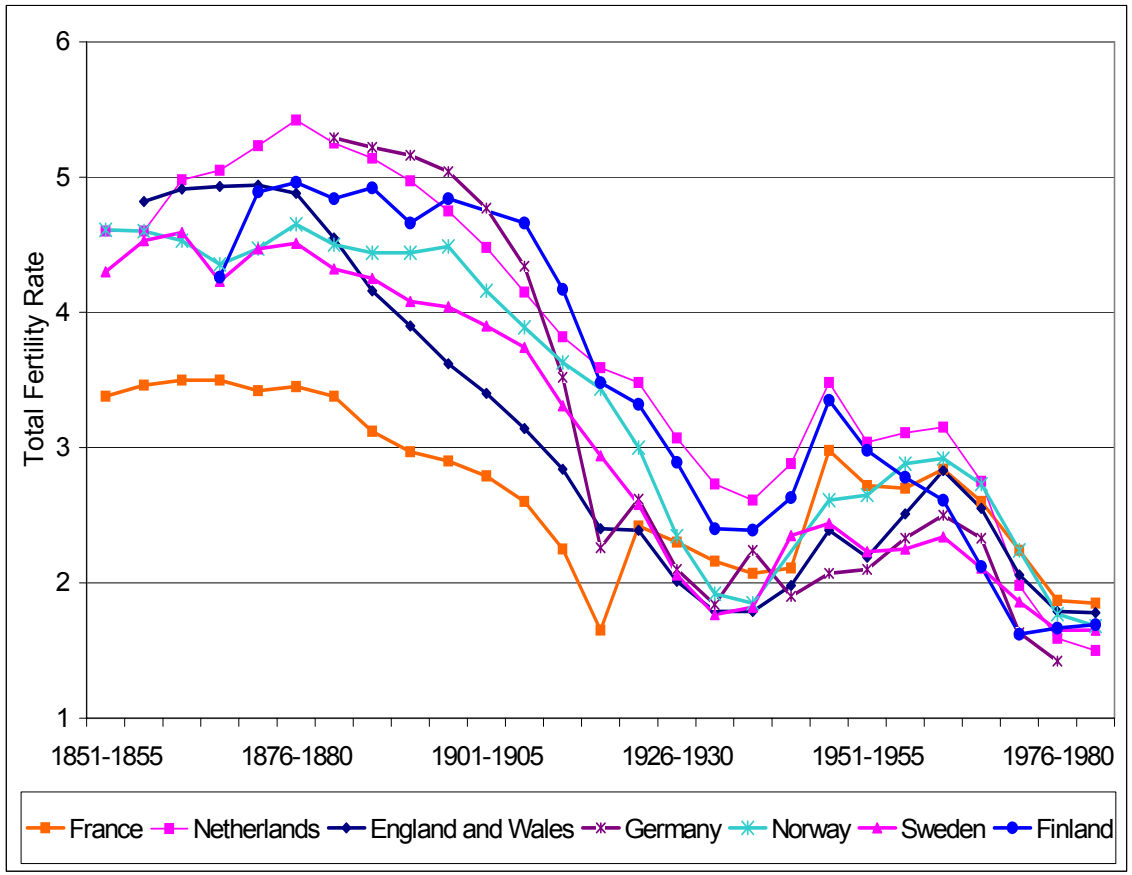


- Net Reproduction Rate = the number of daughters per woman who reach the reproduction age

5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

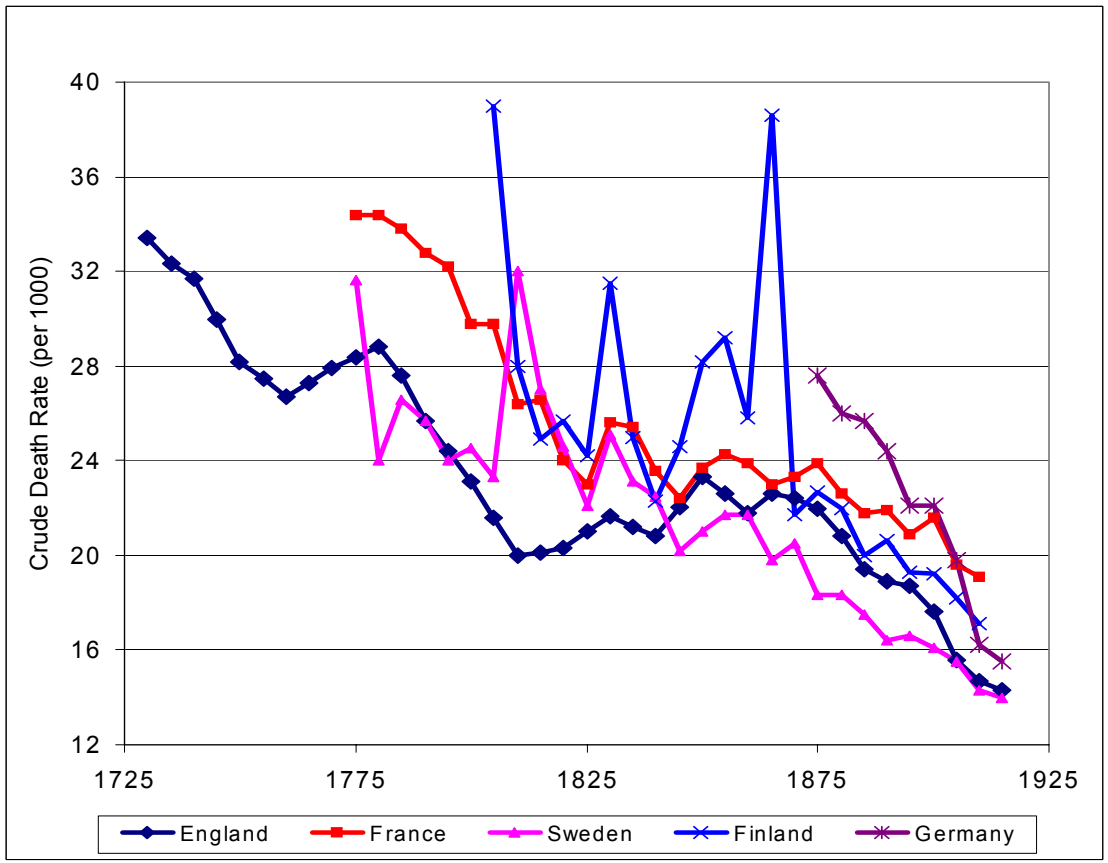
FIGURE 21: Total fertility rates (GALOR [2005])



5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

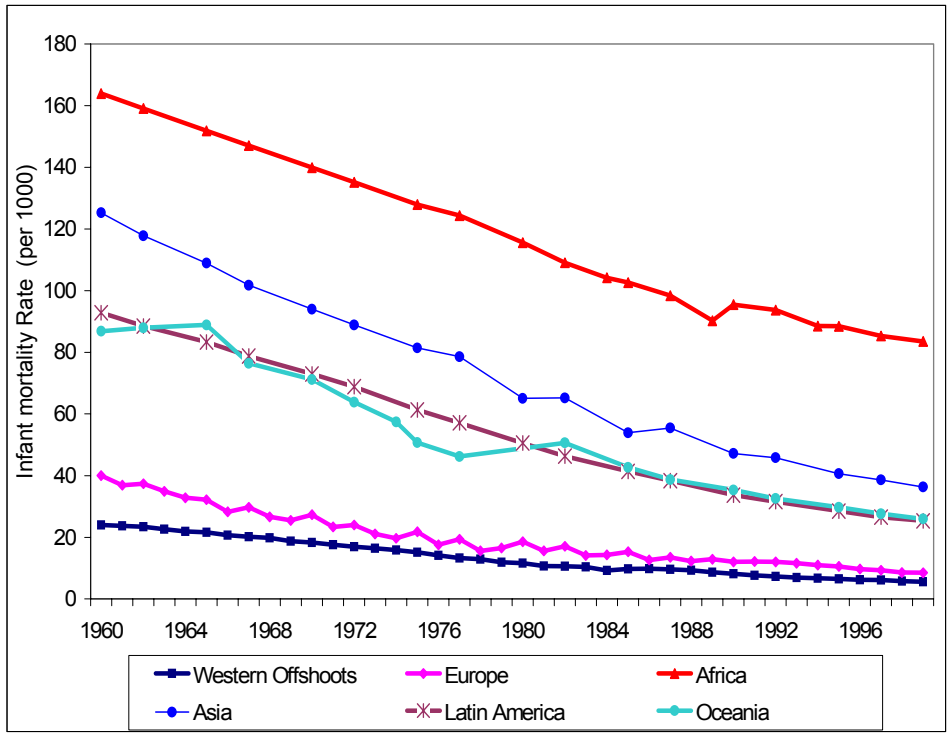
FIGURE 22: Crude death rates (GALOR [2005])



5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

FIGURE 23: Infant mortality rates (GALOR [2005])

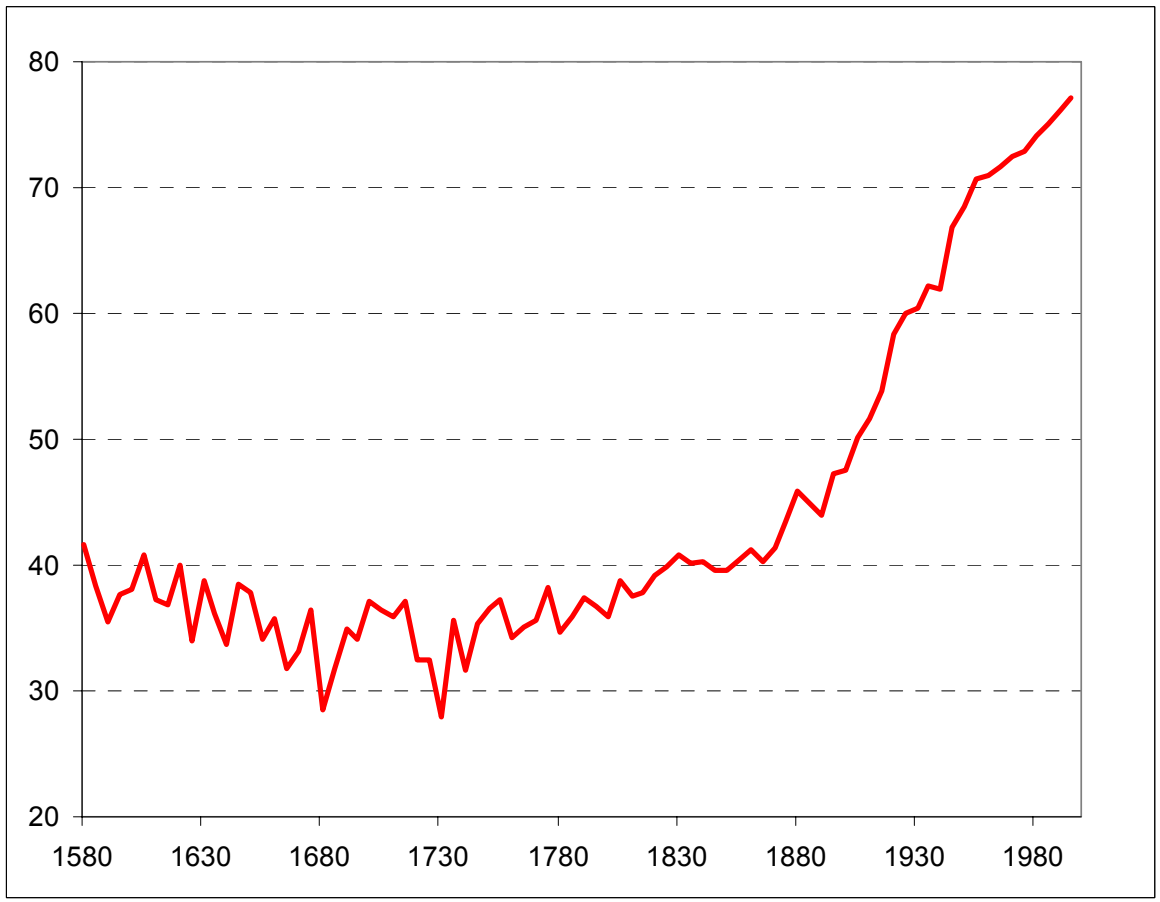


- Infant mortality rate (IMR) is the number of deaths of children less than one year of age per 1000 live births.

5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

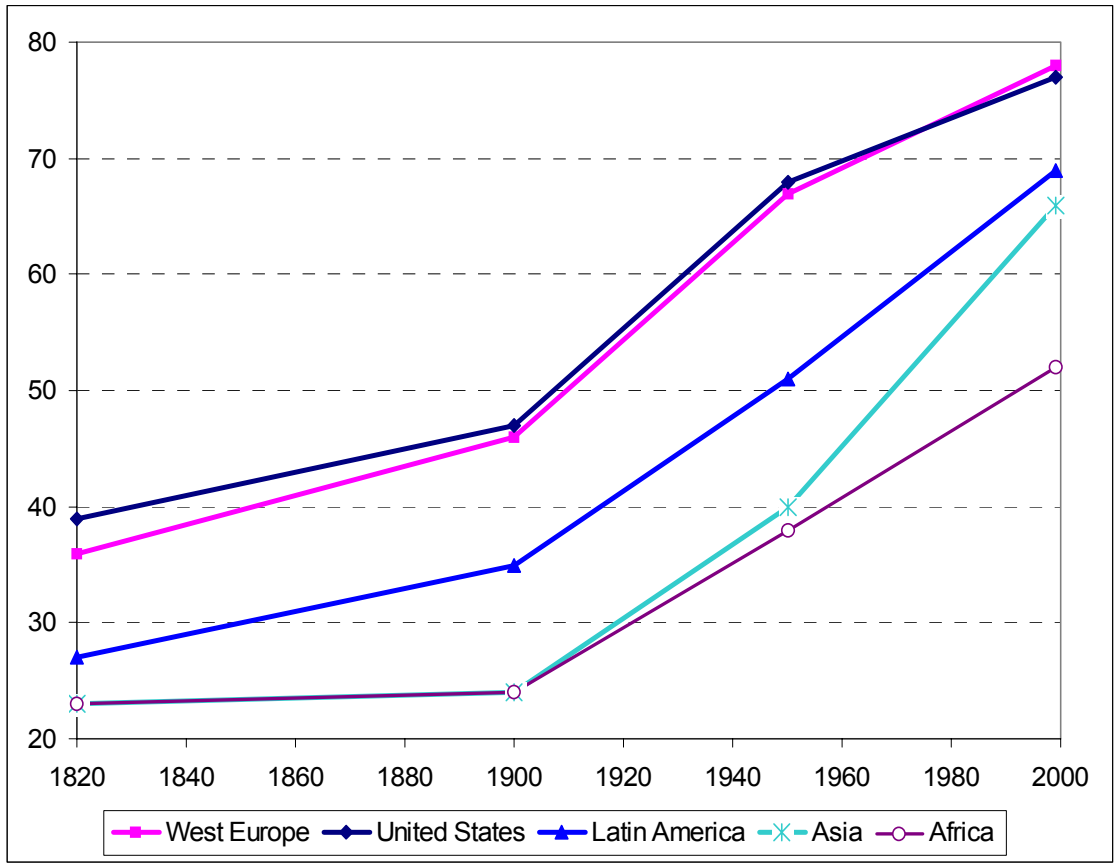
FIGURE 24: Life expectancy in England (GALOR [2005])



5. Demographics and the simple functioning of the Malthusian regime

Some facts, Malthusian regime and after

FIGURE 25: Life expectancy across regions (GALOR [2005])



5. Demographics and the simple functioning of the Malthusian regime

Building blocks

- adapted from CLARK's "Farewell to Alms".
- In the Mathusian regime,

  Demography (1) : the birth rate b is high when income per capita y is high.

  $$b = b\underbrace{(y)}_{+}$$

  Demography (2) : the mortality rate m is low when income per capita y is high.

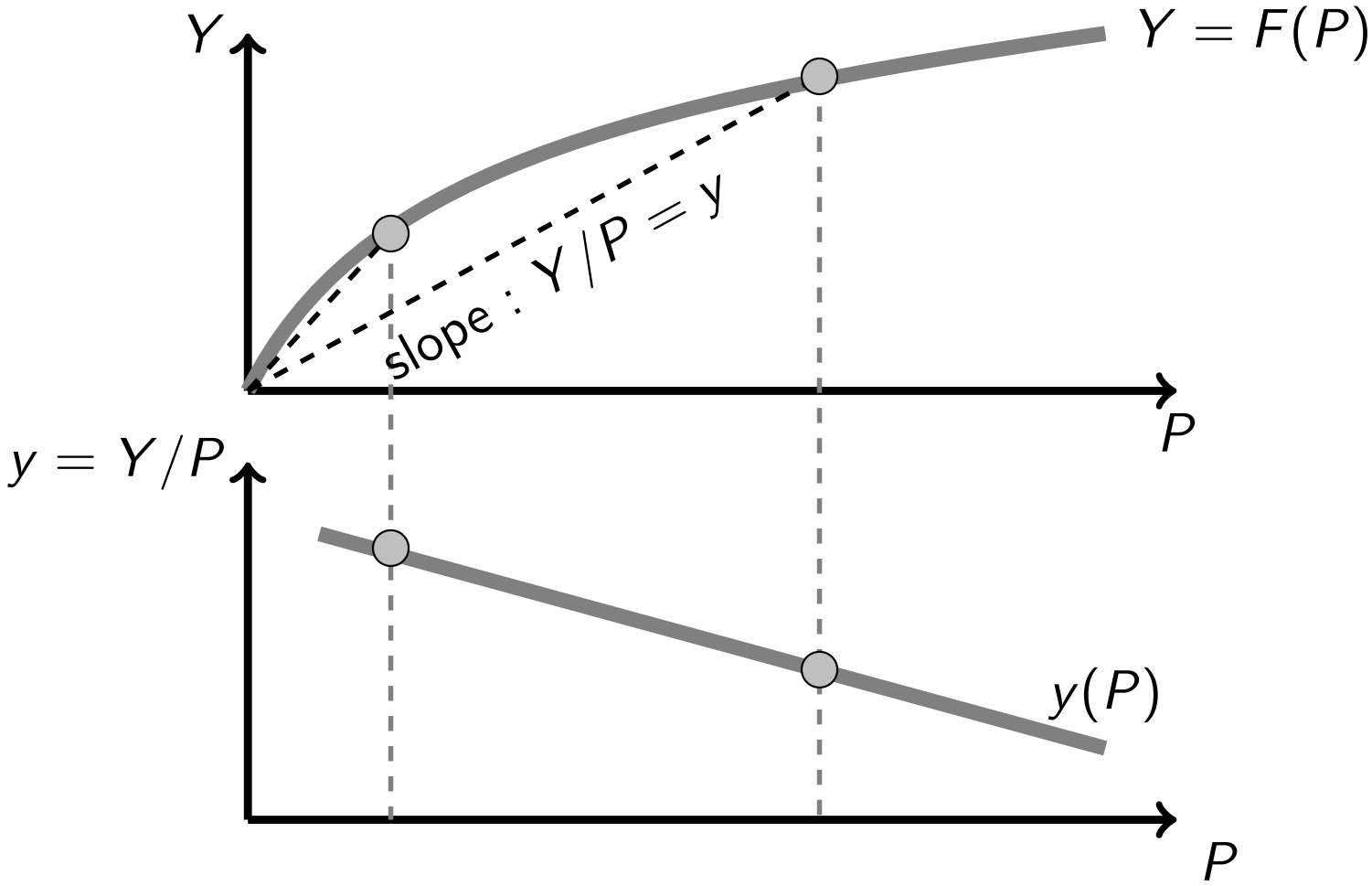
  $$m = m\underbrace{(y)}_{-}$$

  Technology : Population P is low when income per capita y is high.

  $$P = P\underbrace{(y)}_{-}$$

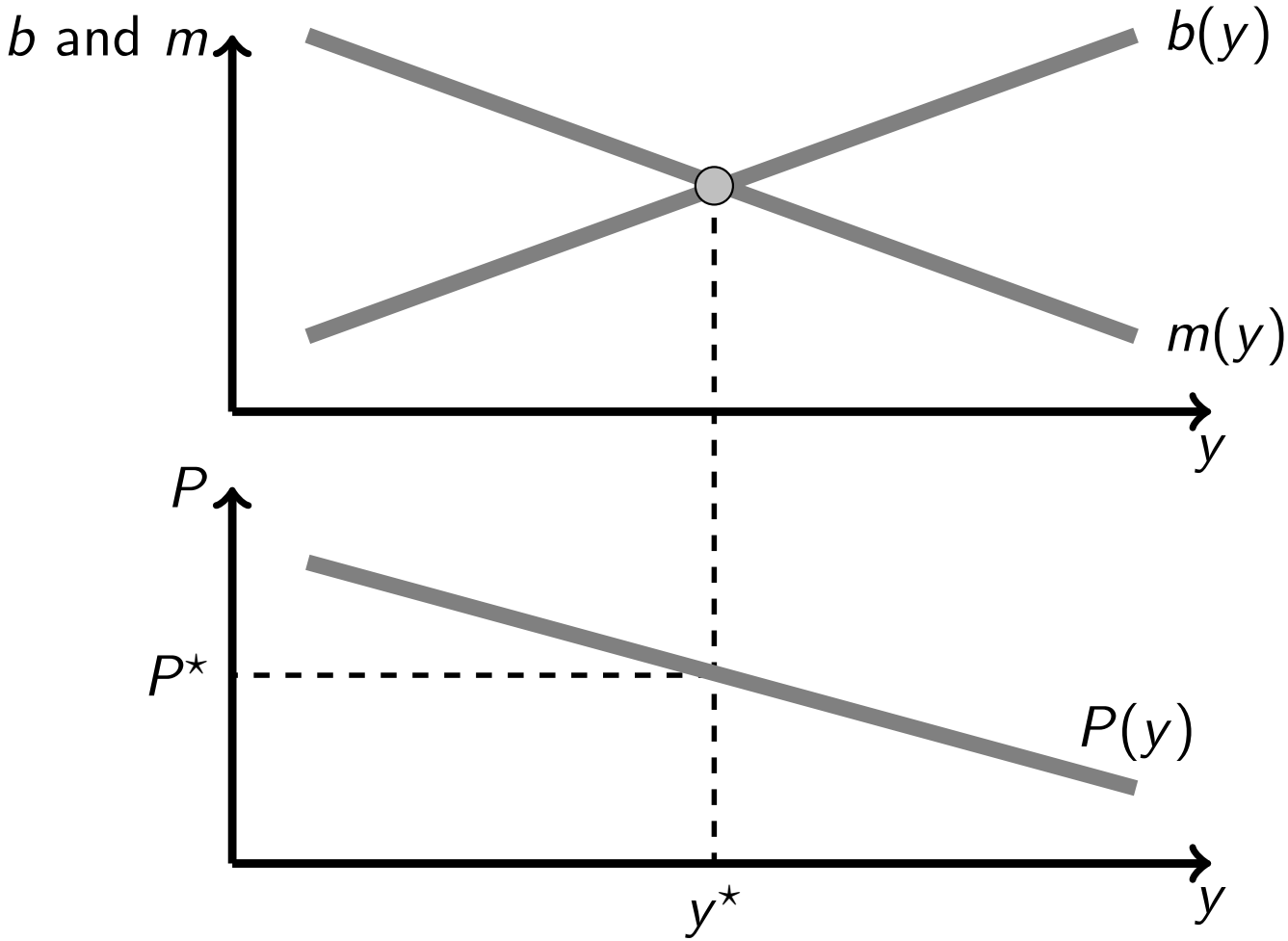
5. Demographics and the simple functioning of the Malthusian regime

FIGURE 26: The technological schedule $P = P(y)$ *(what is shown is the inverse function $y = y(P)$)*



5. Demographics and the simple functioning of the Malthusian regime

FIGURE 27: Long run equilibrium determination



5. Demographics and the simple functioning of the Malthusian regime

Dynamics

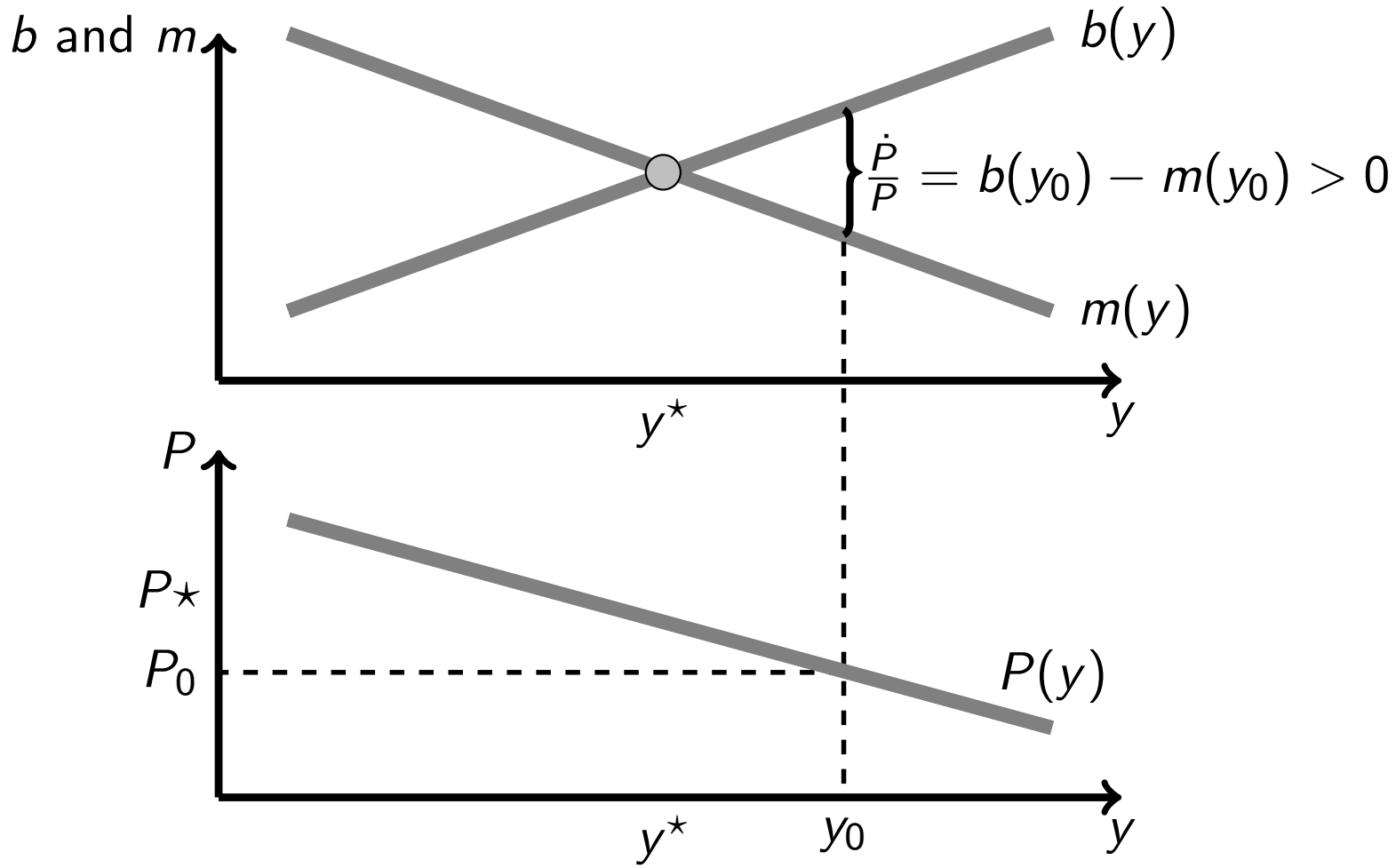
- All the dynamics of the model is given by the population law of motion :

$$\dot{P} = (b - m)P$$

- Let's see what happens if the economy starts from y_0 above $y^\star$.

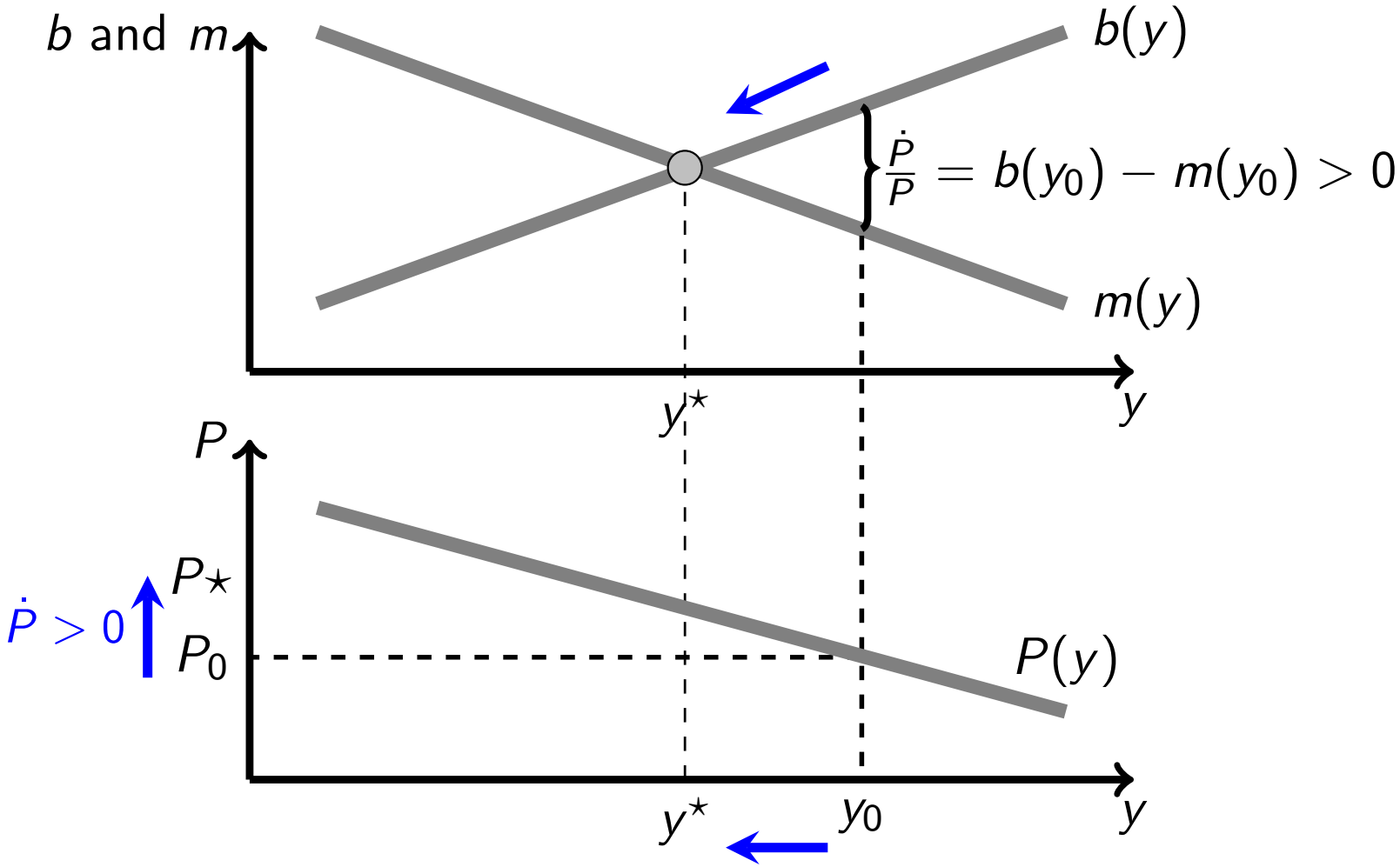
5. Demographics and the simple functioning of the Malthusian regime

FIGURE 28: Dynamics from $y_0 > y^\star$



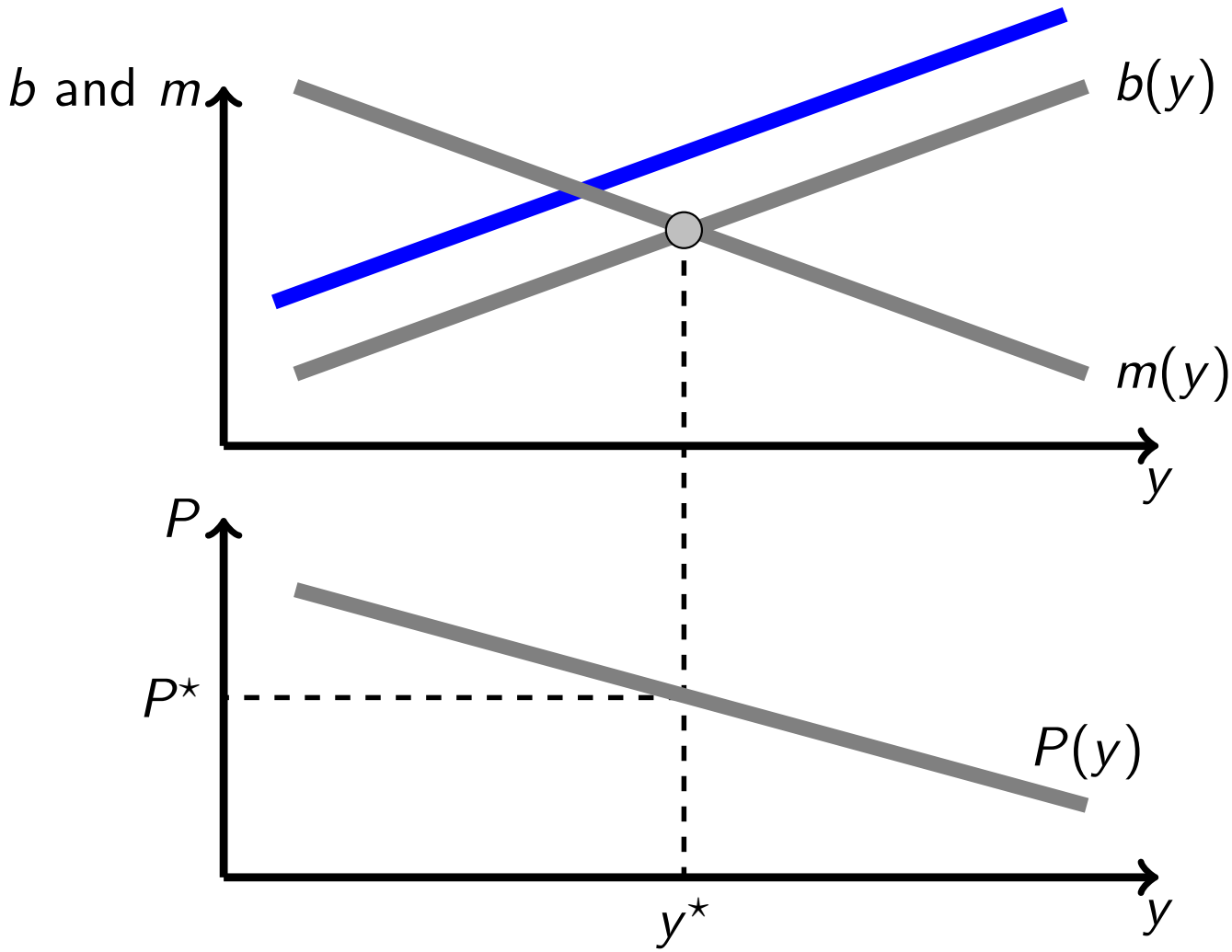
5. Demographics and the simple functioning of the Malthusian regime

FIGURE 29: Dynamics from $y_0 > y^\star$



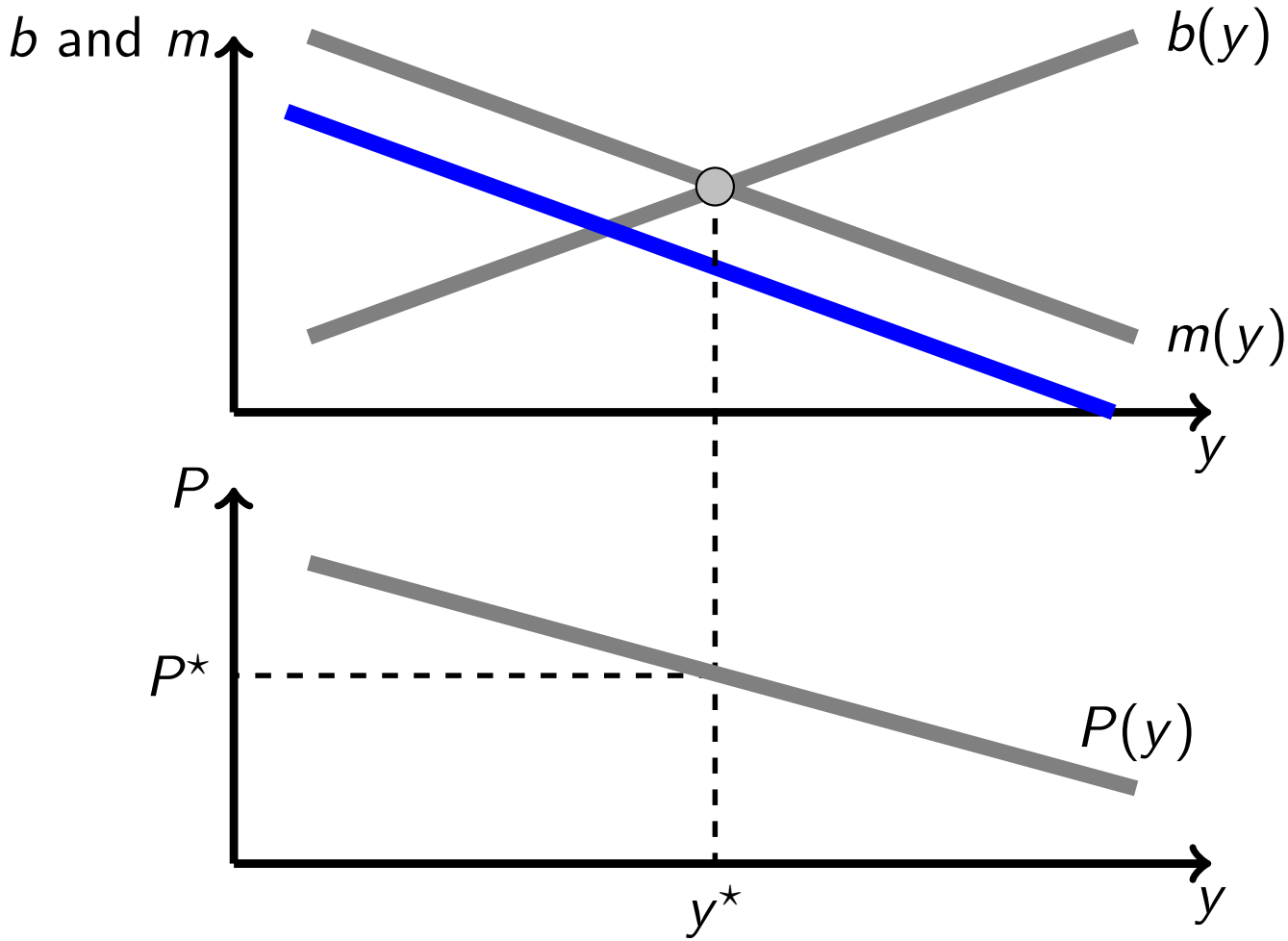
5. Demographics and the simple functioning of the Malthusian regime

FIGURE 30: An increase in the birth rate *(to be completed in class)*



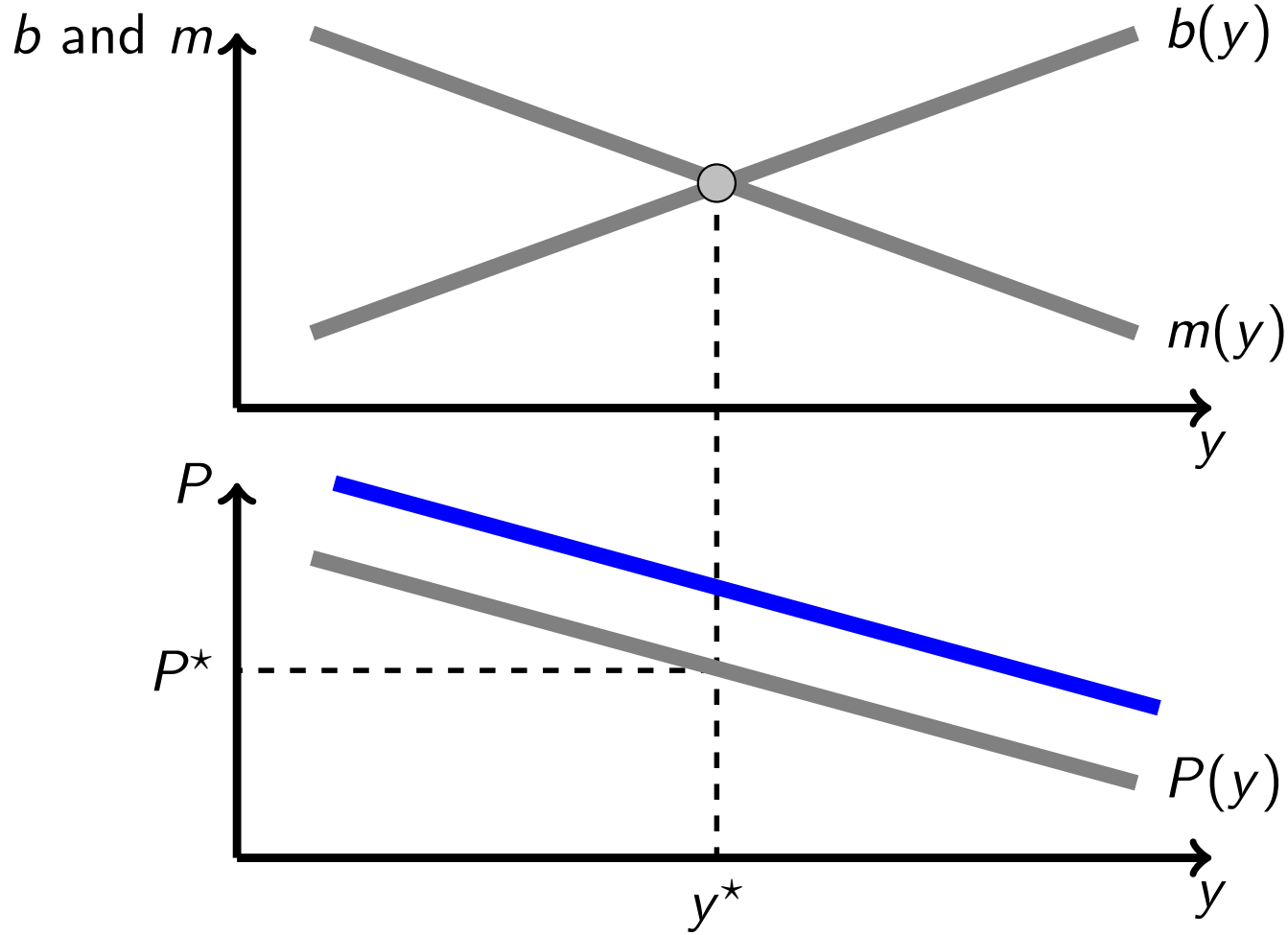
5. Demographics and the simple functioning of the Malthusian regime

FIGURE 31: A decrease in the mortallity rate *(to be completed in class)*



5. Demographics and the simple functioning of the Malthusian regime

FIGURE 32: A technological progress *(to be completed in class)*



6. A (quite) simple mathematical model of life expectancy

Main assumption

- Let us prove that there is an inverse relationship between mortality rate and life expectancy, in a simple model (with some strong assumption).
- Central assumption : for an individual born at time 0, the probability that his or her life length is larger than x is given by e^{-mx}.
- Let us prove that m is indeed the mortality rate, *i.e.* the rate at which people die at each instant in time.
- For that, we need to apply what we know about continuous random variables.

6. A (quite) simple mathematical model of life expectancy

Cumulative distribution function

- Let's assume that for an individual at time 0, the remaining life time, denoted L has a probability $e^{-m\ell}$ to be larger than the value ℓ :

$$P(L > \ell) = e^{-m\ell}$$

- It follows that L is distributed with cumulative function

$$F(\ell) = 1 - P(L > \ell) = 1 - e^{-m\ell}.$$

- What does it mean ? That the probability of that life length is shorter than ℓ is $1 - e^{-m\ell}$

$$F(\ell) = P(L \leq \ell) = 1 - e^{-m\ell}.$$

- Check that when $\ell > 0$ ($\ell < 0$ does not make sense), we have F between 0 and 1, as a probability should be, and that the probability of living an infinite life is 0.

6. A (quite) simple mathematical model of life expectancy

Probability of dying

- Let Δ be a time interval..
- From the point of view of date 0, what is the probability of dying between age ℓ and age $\ell + \Delta$? It is given by

$$\begin{aligned} P(\ell < L < \ell + \Delta) &= F(\ell + \Delta) - F(\ell) \\ &= \left(1 - e^{-m(\ell+\Delta)}\right) - \left(1 - e^{-m\ell}\right) \\ &= e^{-m\ell} - e^{-m(\ell+\Delta)} \end{aligned}$$

- Let's take a first order approximation of $e^{-m(\ell+\Delta)}$ around ℓ, meaning when length of the time elapsed between ℓ and $\ell + \Delta$ becomes small.
- First recall that the first order approximation of any differentiable function $f(x)$ around $f(x_0)$ is

$$f(x) \approx f(x_0) + f'(x_0) \times (x - x_0)$$

6. A (quite) simple mathematical model of life expectancy

Probability of dying

- first order approximation of $e^{-m(\ell+\Delta)}$ around ℓ :

$$e^{-m(\ell+\Delta)} \approx e^{-m\ell} - me^{-m\ell}\Delta$$

- And therefore

$$\begin{aligned} P(\ell < L < \ell + \Delta) &= e^{-m\ell} - e^{-m(\ell+\Delta)} \\ &\approx m\Delta e^{-m\ell} \end{aligned}$$

- Let the time unit be the year. From the point of view of today (that I call date 0, the probability of dying in the year is therefore obtained with $\ell = 0$ and $\Delta = 1$:

$$\begin{aligned} P(0 < L < 1) &= \text{Probability of dying with a year} \\ &= m \end{aligned}$$

6. A (quite) simple mathematical model of life expectancy

Probability of dying

- Notice that the probability of dying in the coming year is m, and does not depends on age (!)
- This is related to the assumption $P(L > \ell) = e^{-m\ell}$
- It is *not* a realistic assumption, but it simplifies the algebra without affecting the qualitative results.

- Let's now compute life expectancy e at birth (which is in the model also life expectancy at *any* age.
- Density of the random variable "life length" is $f(\ell) = F'(\ell) = me^{-m\ell}$
- e is given by

$$e = \int_0^\infty \ell f(\ell) d\ell = \int_0^\infty \ell m e^{-m\ell}$$

- We solve this integral integrating by parts. Let

$$u(\ell) = \ell \quad \text{and} \quad v'(\ell) = me^{-m\ell}.$$

- We have

$$u'(\ell) = 1 \quad \text{and} \quad v(\ell) = -e^{-m\ell}.$$

6. A (quite) simple mathematical model of life expectancy

Life expectancy

- With these notations,

$$\begin{aligned} e &= \int_0^\infty uv' d\ell \\ &= \Big[uv\Big]_0^\infty - \int_0^\infty u'v d\ell \\ &= \Big[-\ell e^{-m\ell}\Big]_0^\infty - \int_0^\infty -e^{-m\ell} d\ell \\ &= 0 + \Big[-\frac{1}{m} e^{-m\ell}\Big]_0^\infty \\ &= \frac{1}{m} \end{aligned}$$

- This shows that life expectancy at birth is equal to the inverse of the mortality rate (given the specific assumption that mortality rate is not age dependent)

7. A simple model of the (very) long economic history

Overview

- Adapted from Strulik & Weisdorf [2008]
- This is a *Dynamic General Equilibrium model with micro foundations*.
- What does it mean ?
  - × Agents maximize utility under budget constraint
  - × They liv more than one periods
  - × Prices are determined by equality of demand and supply on all markets

7. A simple model of the (very) long economic history

Fertility

- Agents live 2 periods (OLG model).
- childhood : consume, at the expense of the parents.
- adulthood : work 1 unit, have children, feed them, consume, and then die.
- In a given period, population is composed of children and adults.
- the period after, adults are dead, children are adults and they have children.
- Simplifying assumption : asexual reproduction : one (single) adult gets n_t children.
- Labor force dynamics : $L_{t+1} = n_t L_t$
- n_t = fertility (no children die)

7. A simple model of the (very) long economic history

Preferences

- Adults like manufacturing goods m_t and children n_t.
- quasi-linear utility : $u_t = m_t + \gamma \log n_t$.
- Price of m is 1 by normalization, price of food is p.
- For simplicity, people consume only when young. They need $a = 1$ unit of food.
- Therefore, 1 child costs $a = 1$ unit of food.
- The budget constraint of an adult is therefore $p_t n_t + m_t = w_t$
- w_t is an adult income

7. A simple model of the (very) long economic history

The demand for children

$$\max_{n_t, m_t} \quad m_t + \gamma \log n_t$$
$$s.t. \quad p_t n_t + m_t = w_t$$

- solution (*details in class*) : $n_t = \frac{\gamma}{p_t}$
- Discussion
  - × Strong preferences for children : income does not affect n (zero income elasticity)
  - × Only the price of food affect fertility decisions.
  - × But there is an indirect effect of growth (and therefore income) on fertility :
    - Agric. productivity growth $\rightsquigarrow$ p decreases $\rightsquigarrow$ fertility increases.
    - Manuf. productivity growth $\rightsquigarrow$ p increases $\rightsquigarrow$ fertility decreases.
- Malthus preventive check hypothesis : scarce agricultural goods (high p) $\rightsquigarrow$ low fertility.

7. A simple model of the (very) long economic history

Production

- All technologies are constant returns.
- Food : $Y_t^A = A_t^\epsilon (L_t^A)^\alpha X^{1-\alpha}$
- X is land, assumed to be in fixed supply, and normalized to $X = 1$; L^A is agricultural labor and A is productivity on agriculture. ϵ and α are between 0 and 1.
- Manufacturing : $Y_t^M = M_t^\epsilon L_t^M$
- Assume for simplicity that land is free disposal (land rent is zero)

7. A simple model of the (very) long economic history

Growth

- Growth is endogenous, and comes from technological progress $A^{t+1} - A_t$ and $M_{t+1} - M_t$.
- We assume *Leaning-by-doing* :
  - $A_{t+1} - A_t = Y_t^A$
  - $M_{t+1} - M_t = Y_t^M$
- Note that this comes as an externality : nobody gets money for the increase in A and M.
- No capital for simplicity.
- Notation : for a variable Z, $g_t^Z = \frac{Z_{t+1} - Z_t}{Z_t}$
- Therefore,

$$\begin{aligned} g_t^A &= A_t^{\epsilon-1}(L_t^A)^{\alpha} \\ g_t^M &= M^{\epsilon-1} L_t^M \end{aligned}$$

7. A simple model of the (very) long economic history

Dynamic General (Macroeconomic) Equilibrium

- What is an equilibrium in such an economy ?
- In period t, given L_t and productivities A_t and M_t :
  - × Adults work 1 unit and choose optimally fertility (=food demand) and manuf. goods demand given prices and their income.
  - × Adults choose to work as farmers or manufacturers.
  - × Labor market clears.
  - × Food and manufacturing goods markets clear.
- Solving for the general equilibrium means to find p_t, w_t, n_t, Y_t^A, Y_t^M.
- Once we have solved, we can compute L_{t+1} and productivities A_{t+1} and M_{t+1} $\rightsquigarrow$we can move on and solve for period $t+1$
- Once we are given L_0, A_0 and M_0, we can solve for the whole history of human activity : everything is endogenous : production, population and technology.
- This is a model of *Unified Growth*.

- Equilibrium on the food market :
  - × demand : $n_t L_t$
  - × supply : $A_t^{\epsilon}(L_t^A)^{\alpha}$
  - × Let $\theta_t = \frac{L_t^A}{L_t}$ be the share of total labor devoted to agriculture, food supply = food demand implies

$$\theta_t = \left(\frac{n_t L_t^{1-\alpha}}{A_t^{\epsilon}} \right)^{\frac{1}{\alpha}}$$

7. A simple model of the (very) long economic history

Dynamic General (Macroeconomic) Equilibrium

- Equilibrium on the labor market :
  - The wage has to be the same in the two sectors.
  - In each sector, the only cost is therefore labor, so that $w_t^A = \frac{p_t Y_t^A}{L_t^A}$ and $w_t^M = \frac{Y_t^M}{L_t^M}$
  - The labor market equilibrium condition implies that the relative price of food adjusts, so that farmers and manufacturers earn the same income=

$$w_t^A = w_t^M$$

  - which gives (*details in class*)

$$p_t = \frac{M_t^{\alpha\epsilon}(\gamma L_t)^{1-\alpha}}{A_t^{\epsilon}}$$

  - Given that $p_t = \frac{\gamma}{n_t}$, we obtain

$$n_t = \gamma^{\alpha} \frac{A_t^{\epsilon}}{M_t^{\alpha\epsilon} L_t^{1-\alpha}}$$

7. A simple model of the (very) long economic history

Model dynamics

- The model dynamics is then computed as follows :
- Take an initial period 0. (L_0, A_0, M_0) are given.
- Fertility (and therefore population growth) is given by

$$n_0 = \gamma^{\alpha} \frac{A_0^{\epsilon}}{M_0^{\alpha\epsilon} L_0^{1-\alpha}}$$

- The equilibrium allocation of labor between agriculture and manufacturing is then given by

$$\theta_0 = \left(\frac{n_0 L_0^{1-\alpha}}{A_0^{\epsilon}} \right)^{\frac{1}{\alpha}}$$

so that $L_0^A = \theta_0 L_0$ and $L_0^M = (1 - \theta_0) L_0$

7. A simple model of the (very) long economic history

Model dynamics

- Then production in each sectors are given by $Y_0^A = A_0^{\epsilon}(L_0^A)^{\alpha}$ and $Y_0^M = M_0^{\epsilon} L_0^M$.
- Form the learning-by-doing mechanism, we obtain the period 1 level of productivity : $A_1 = A_0 + Y_0^A$ and $M_1 = M_0 + Y_0^M$
- Finally, $L_1 = n_0 L_0$
- We have computed $(L_1, A_1, M_1) \rightsquigarrow$ we can now move on and compute the other equilibrium variables of period 1, etc...

7. A simple model of the (very) long economic history

Balanced Growth Paths

- BGP are path along which variables grow at constant rate (possibly zero).
- We have shown that $g_t^A = A_t^{\epsilon-1}(L_t^A)^\alpha$
- along a BGP, $A_t^{\epsilon-1}(L_t^A)^\alpha$ must be constant, meaning that $(1+g_A) = (1+g_L)^{\alpha/(1-\epsilon)}$.
- Similarly, $g_t^M = M^{\epsilon-1}L_t^M$ implies $(1+g_M) = (1+g_L)^{1/(1-\epsilon)}$.
- From

$$n_t = \gamma^\alpha \frac{A_t^\epsilon}{M_t^{\alpha\epsilon} L_t^{1-\alpha}}$$

we get that the growth rate of fertility (i.e. of the population growth factor) is

$$\frac{n_{t+1}}{n_t} = \frac{(1+g_t^A)^\epsilon}{(1+g_t^M)^{\alpha\epsilon} n_t^{1-\alpha}}$$

- Using $n_t = (1+g_t^L)$ and the two previous equations, we get :

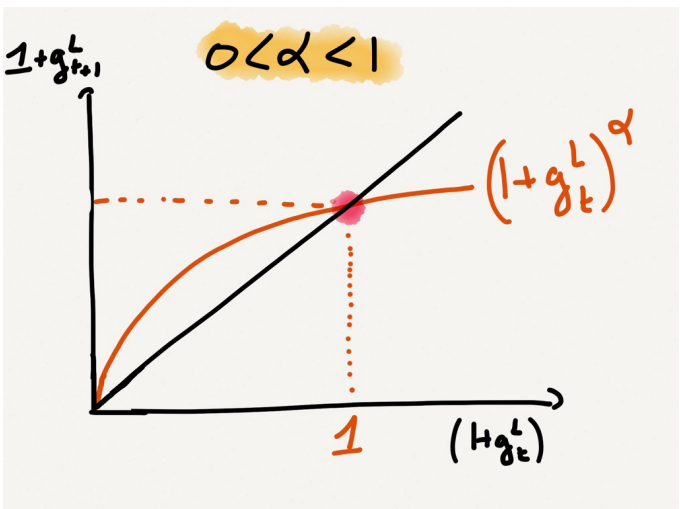
$$(1+g_{t+1}^L) = (1+g_t^L)^\alpha$$

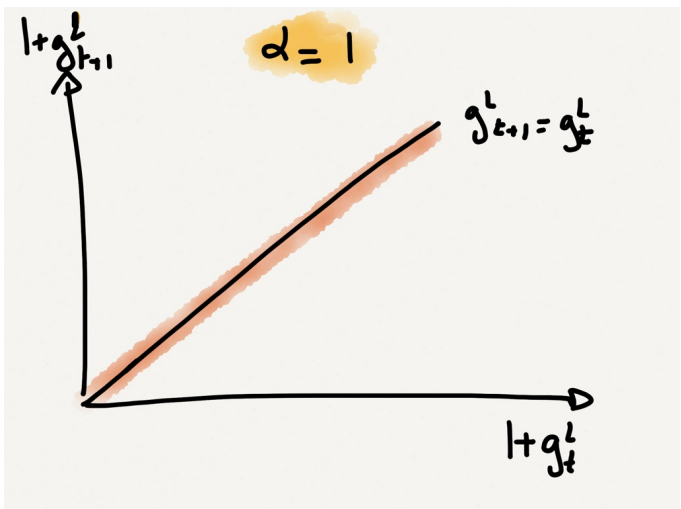
7. A simple model of the (very) long economic history

Balanced Growth Paths

$$(1 + g^L_{t+1}) = (1 + g^L_t)^\alpha$$

- This is a quite simple difference equation.
- Along a BGP, g_L is constant.
- Two possible solutions :
  - × if $\alpha < 1$, $g^L = 0$: population is constant in the long run
  - × if $\alpha = 1$ (land is not needed to produce food) : any initial g^L is a solution (imploding or exploding growth). But this is a not a interesting/realistic case.





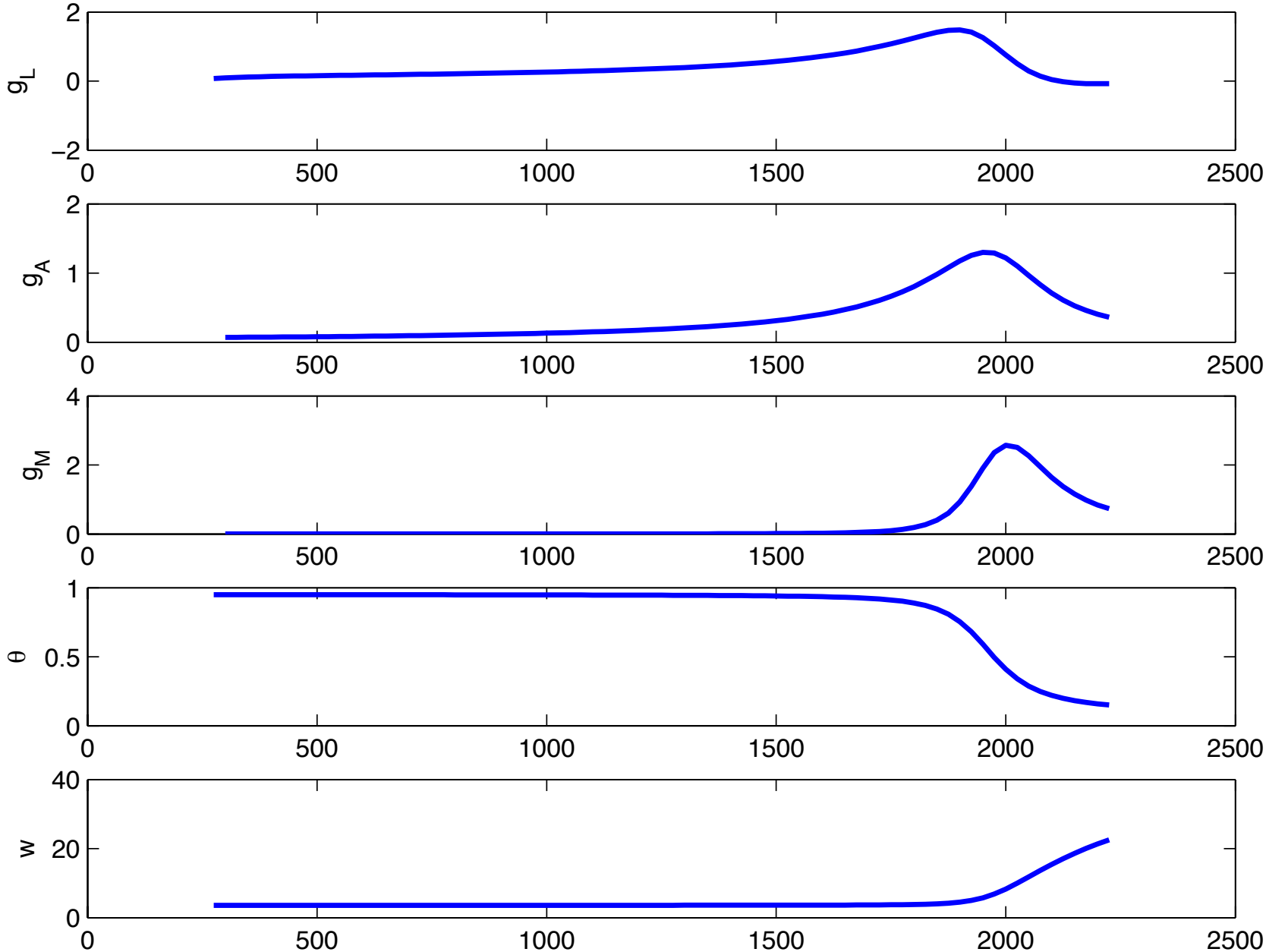
7. A simple model of the (very) long economic history

An numerical example of the full dynamics

- We can use a spreadsheet to compute the full dynamics of the economy starting from some initial conditions and for some values of the parameters.
- For this purpose, we need to slightly modify the model (theoretically unimportant, but useful for the simulations)
- $Y_t^A = A_t^\epsilon (L_t^A)^\alpha X^{1-\alpha} \rightsquigarrow Y_t^A = \mu A_t^\epsilon (L_t^A)^\alpha X^{1-\alpha}$
- $Y_t^M = M_t^\epsilon L_t^M \rightsquigarrow Y_t^M = \delta M_t^\phi L_t^M$
- In that case, it is easy to show that $n_t = \mu \left(\frac{\gamma}{\delta}\right)^\alpha \frac{A_t^\epsilon}{M_t^{\alpha\phi} L_t^{1-\alpha}}$ and that $\theta_t = \left(\frac{n_t L_t^{1-\alpha}}{\mu A_t^\epsilon}\right)^{\frac{1}{\alpha}}$
- Parameters : $\alpha = 0.8$, $\epsilon = 0.45$, $\phi = 0.3$, $\mu = 0.5$, $\delta = 1.5$, $\gamma = 3.4$ and initial values are $A_0 = 0.8$, $M_0 = 18.15$ and$L_0 = 0.014$.

A simple model of the (very) long economic history

An numerical example of the full dynamics



8. Summary

- We have sketched the three phases of the economic history if the world : The Malthusian regime, the Industrial Revolution and the Great Divergence
- We have studied a demographic model of the Malthusian regime
- We have studied a mathematical model of life expectancy under the simplifying assumption that death is not age-dependent
- We have studied a dynamic general equilibrium model of Unified Growth Theory.
- This model endogenizes demographics and technical progress

9. References

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