FRANCK PORTIER
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M1 MACRO

GROUP A

Session 3

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

Gen Eq. with a market for savings

$$\max \log c_1 + \beta \log c_2$$

$$BC_1 \quad c_1 + s = y_1$$

choose optimally c_1, c_2, s

$$BC_2 \quad c_2 = y_2 + (1+r_2)\, s$$

price to be determined in gen. eq.

$$BC_1: \quad s = y_1 - c_1$$

$$BC_2: \quad c_2 = y_2 + (1+r)(y_1 - c_1)$$

$\Longleftrightarrow$

Intertemporal Budget Constraint (IBC)

$$(1+r)c_1 + c_2 = \underbrace{(1+r)}_{R} y_1 + y_2$$

$\Longleftrightarrow$

IBC $$c_1 + \frac{c_2}{1+r_2} = y_1 + \frac{y_2}{1+r_2}$$

sum of disc. expenditures

sum of discounted endowments

$\Rightarrow$ Nerpb $\quad \max \log c_1 + \beta \log c_2$

s.t. $c_1 + \frac{c_2}{1+r_2} = y_1 + \frac{y_2}{1+r}$

Take the previous compet. eq.

$$\begin{cases} p_1 = 1 \\ p_2 = \beta^{y_1/y_2} \end{cases}$$

if $\frac{1}{1+r} = p_2$,

then r is the equilibrium interest rate. $r = \frac{1}{p_2} - 1 = \frac{y_2}{\beta y_1} - 1$

Optimality condition

$$\frac{1/c_2}{1/c_1} = \frac{p_2}{p_1}$$

$$\Leftrightarrow \quad \frac{1}{c_1} = \frac{p_1}{p_2}\,\beta\,\frac{1}{c_2}$$

$$\frac{1}{c_1} = (1+r)\,\beta\,\frac{1}{c_2}$$

Intertemp Euler Equilibrium

Introducing uncertainty

[optimal behavior of an agent given prices]

Assume Y_2 is uncertain

$$Y_2 = \begin{cases} E(Y_2) - 1 & \text{with prob } 1/2 \\ E(Y_2) + 1 & \text{with p } 1/2 \end{cases}$$

check

$$E(y_2) = \frac{1}{2}\left[E(y_2)+1\right] + \frac{1}{2}\left[E(y_2)-1\right]$$

$$= 2 \cdot \frac{1}{2} E(y_2) + \frac{1}{2} - \frac{1}{2}$$

$$= E(y_2)$$

It makes sense to assume that
with uncertainty, agents
maximize Expected utility

$$\max_{c_1, s, c_2} \; u(c_1) + \beta E_1[u(c_2)]$$

$$\text{s.t.} \quad \begin{cases} c_1 + s = y_1 \\ c_2 = y_2 + (1+r)s \end{cases}$$

IBC $\quad c_1 + \frac{c_2}{1+r_2} = y_1 + \frac{y_2}{1+r_2}$

$$c_2 = (1+r_2)y_1 + y_2 - (1+r_2)c_1$$

$$\max_{c_1} \log c_1 + \beta E_1 \log\left[(1+r_2)y_1 + y_2 - (1+r_2)c_1\right]$$

$$\Leftrightarrow \max_{c_1} \log c_1 + \beta \left[\begin{array}{l} \tfrac{1}{2}\left(\log\left[(1+r_2)y_1 + E(y_2) + 1 - (1+r_2)c_1\right]\right. \\ \left. + \tfrac{1}{2}\left(\log\left[(1+r_2)y_1 + E(y_2) - 1 - (1+r_2)c_1\right]\right)\right] \end{array}$$

FOC $$\frac{1}{c_1} = \beta(1+r) E_1\left[\frac{1}{c_2}\right]$$

stochastic Euler equation.

$$c_2 = \begin{cases} \text{high} & \text{prob } 1/2 \quad c_{2H} \\ \text{low} & \text{— } 1/2 \quad c_{2L} \end{cases}$$

→ with risk: $E[c_2]$, $E[y_2]$

Without risk, this is what we did before $Y_2 = E(Y_2)$

$$\frac{1}{c_1} = (1+r)\beta \frac{1}{E(c_2)}$$

same c_1?

with risk

$$\frac{1}{c_1} = (1+r)\beta E\left[\frac{1}{c_2}\right]$$