FRANCK PORTIER
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M1 MACRO

GROUP B

Session 3

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

Two period model with saving

$$\max u = \log c_1 + \beta \log c_2$$

BC period 1: $c_1 + s = y_1$

period 2: $c_2 = y_2 + s(1+r)$

r is taken as given
= interest rate

Intertemporal Budget Constraint (IBC)

$$\begin{cases} c_1 + s = y_1 \\ c_1 = y_2 + (1+r)s \Rightarrow s = \dfrac{c_2 - y_2}{1+r} \end{cases}$$

$$c_1 + \frac{c_2 - y_2}{1+r} = y_1 \quad (\cdot)$$

$$\boxed{c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}} \quad \text{IBC}$$

opt. c_1 and c_2 of the household

$$\max \log c_1 + \beta \log c_2$$

$$\text{s.t.} \quad \boxed{c_1 + \frac{1}{1+r} c_2 = y_1 + \frac{1}{1+r} y_2}$$

$$p_1 c_1 + p_2 c_2 = p_1 y_1 + p_2 y_2$$

$$\hookrightarrow \quad \boxed{c_1 + \frac{p_2}{p_1} c_2 = y_1 + \frac{p_2}{p_1} y_2}$$

in eq, $\frac{1}{1+r} = p_2 / p_1$

eq. price $P_1 = 1$, $P_2 = \beta \frac{\gamma_1}{\gamma_2}$

eq interest rate: $\frac{1}{1+r} = \beta \frac{\gamma_1}{\gamma_2}$

$$r = \frac{1}{\beta}\frac{\gamma_2}{\gamma_1} - 1$$

eq. interest rate

check that $c_1 = y_1$ for eq. r.

Stochastic endowment

y_1 given

$$y_2 = \begin{cases} \bar{y}+1 = y_L & \text{prob } \frac{1}{2} \\ \bar{y}-1 = y_H & \text{prob } \frac{1}{2} \end{cases}$$

$$E[y_2] = \bar{y}$$

- Agents maximize **Expected utility**.

Consider the lottery: $c_2 = \begin{cases} c_{2L} & 1/2 \\ c_{2H} & 1/2 \end{cases}$

the utility of this lottery is

in $E\,u(c_2) = \frac{1}{2}u(c_{2L}) + \frac{1}{2}u(c_{2H})$

[This is an assumption]

Problem of a household

$$\max \log c_1 + \beta E \log c_2$$

$$\hookrightarrow = \log c_1 + \beta \left[\frac{1}{2} \log c_{2L} + \frac{1}{2} \log c_{2H} \right]$$

BC $c_1 + s = y_1$

$$c_2 = y_2 + (1+r)s$$

$$\hookrightarrow \begin{cases} c_{2L} = y_{2L} + (1+r)s \\ c_{2H} = y_{2H} + (1+r)s \end{cases}$$

$$\max \log c_1 + \beta E \log c_2$$

$$s.t \quad c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

$$\max \log c_1 + \beta E \log \left[(1+r)(y_1 - c_1) + y_2 \right]$$

FOC/

$$\boxed{\frac{1}{c_1} = \beta(1+r) E\left[\frac{1}{c_2}\right]}$$

$$c_1 + \frac{c_2}{1+r_2} = y_1 + \frac{y_2}{1+r_2}$$

↳ means

$$\begin{cases} c_1 + \frac{c_{2L}}{1+r_L} = y_1 + \frac{y_{2L}}{1+r_2} \\ c_1 + \frac{c_{2H}}{1+r_H} = y_1 + \frac{y_{2H}}{1+r_2} \end{cases}$$