FRANCK PORTIER
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M1 MACRO

GROUP B

Session 4

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

I need to solve

$$\frac{1}{c_1} = \beta(1+m) E \frac{1}{c_2}$$ ←

Remark E = mathematical expectation

= linear operation

[$E(aX+b) = aE(X)+b$]

but $E\left[\frac{1}{X}\right] \neq \frac{1}{E(X)}$

Assume $\beta(1+r) = 1$

Euler eq: $\frac{1}{c_1} = E\left[\frac{1}{c_2}\right]$

- let us consider 2 situations

① stochastic: $y_2 = \begin{cases} y_L & \text{prob } 1/2 \\ y_H & - \ 1/2 \end{cases}$ $\quad y_L < y_H$

$c_2 = y_2 + (1+r)S = \begin{cases} c_L & \text{prob } 1/2 \\ c_H & - \ 1/2 \end{cases}$

(2) Deterministic

The agent gets $E(y_2)$ for sure

$$E[y_2] = \frac{1}{2} y_H + \frac{1}{2} y_L$$

so that

$$c_2 = E(c_2) = \frac{1}{2} c_H + \frac{1}{2} c_L$$

FOC $\quad \frac{1}{c_1} = \underbrace{\beta(1+r)}_{1} \frac{1}{c_2} \Leftrightarrow \frac{1}{c_1} = \frac{1}{E(c_2)}$

- Stochastic world

$$\frac{1}{c_1} = E\frac{1}{c_2}$$

- Deterministic world

$$\frac{1}{c_1} = \frac{1}{Ec_2}$$

How the choice of c_1 today is affected by uncertainty?

— = deterministic

— = stochastic

$E\left[\frac{1}{c_a}\right] = \frac{1}{c_1}$

$\frac{1}{c_2}$

$\frac{1}{c_1} = \frac{1}{E(c)}$

$\frac{1}{c_H}$

$\frac{1}{c}$

c_L $E(c)$ c_H c_2 D

$\frac{1}{c_1}$ with uncert. $>$ $\frac{1}{c_1}$ with certainty

$\Leftrightarrow$ c_1 uncert. $<$ c_1 certainty

$S = \gamma_1 - c_1$

$S_{uncert.} > S_{certainty}$

PRECAUTIONARY SAVING

2-period model with labor supply

[not general equilibrium]

$$U = \log c_1 + \beta \log c_2 + \log(1+N_1) + \rho \log(1-N_2)$$

1 unit of time available per period

N_1 = working time

$1-N_1$ = leisure time $\quad , 0 \leq N_1 < 1$

$$IBC \cdot c_1 + \frac{c_2}{R} = w_1 N_1 + \frac{w_2 N_2}{R}$$

$$R = 1 + \pi$$

$$\mathcal{L} = \log c_1 + \rho \log c_2 + \log(1 - N_1) + \rho \log(1 - N_2)$$
$$+ \lambda \left[w_1 N_1 + \frac{w_2 N_2}{R} - c_1 - \frac{c_2}{R} \right]$$

choose optimally: c_1, c_2, N_1, N_2

$\lambda > 0$

FOC

$$\partial/\partial c_1 : \frac{1}{c_1} = \lambda$$

$$\partial/\partial c_2 : \beta/c_2 = \lambda/R$$

$$\Rightarrow \boxed{c_1 = c_2 = c}$$

$$\partial/\partial N_1 : \frac{1}{1-N_1} = \lambda w_1 \Rightarrow 1 - N_1 = \frac{c}{w_1}$$

$$\Leftrightarrow \boxed{N_1 = 1 - \frac{c}{w_1}}$$

$$\partial/\partial N_2 : \cancel{\beta/1-N_2} = \cancel{\lambda \frac{w_2}{R}}$$

$$\boxed{N_2 = 1 - \frac{c}{w_2}}$$

Assume $\beta R = 1$

plug it back in the $\pm$BC

$$C + \frac{C}{R} = W_1\left(1 - \frac{C}{W_1}\right) + \frac{W_2}{R}\left(1 - \frac{C}{W_2}\right)$$

$$\Leftrightarrow \left(1 + \frac{1}{R}\right) C = W_1 + \frac{W_2}{R} - C - \frac{C}{R}$$

$$- C\left(1 + \frac{1}{R}\right)$$

$$\boxed{C = \frac{1}{2}\,\frac{R}{1+R}\left(W_1 + \frac{W_2}{R}\right)} = C_1 = C_2$$

$$N_1 = 1 - \frac{c}{w_1} = 1 - \frac{1}{2}\frac{R}{1+R}\left(1 + \frac{w_2}{w_1}\frac{1}{R}\right)$$

$$N_2 = 1 - \frac{c}{w_2} = 1 - \frac{1}{2}\frac{R}{1+R}\left(\frac{w_1}{w_2} + \frac{1}{R}\right)$$

(1) Temporary wage increase

$$\tilde{w}_1 = \lambda w_1 \qquad \lambda > 1$$

$N_1 \nearrow$, $N_2 \searrow$

(2) Permanent wage increase

$$\tilde{w}_1 = \lambda w_1 \,, \tilde{w}_2 = \lambda w_2 \quad \lambda > 1$$

No change in labor supply