FRANCK PORTIER
2016-2017

M1 MACRO
GROUP A
Session 8

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

Intertemporal budget constraint

(starting in t)

$$A_{t+1} + C_t = (1+r)A_t + Y_t$$

$$A_{t+1} = (1+r)A_t + Y_t - C_t$$

$$A_t = \frac{1}{1+r}\left(A_{t+1} + C_t - Y_t\right) \qquad \text{in } t$$

$$A_{t+2} = \frac{1}{1+r}\left(A_{t+2} + C_{t+1} - Y_{t+1}\right) \qquad \text{in } t+1$$

$$= \frac{1}{1+r}\left(\left[\frac{1}{1+r}\left(A_{t+3} + C_{t+2} - Y_{t+2}\right)\right] + C_{t+1} - Y_{t+1}\right)$$

$$A_{t+1} = \left(\frac{1}{1+r}\right)^2 A_{t+2} + \left(\frac{1}{1+r}\right)^2 (C_{t+2} - Y_{t+2}) + \frac{1}{1+r}(C_{t+1} - Y_{t+1})$$

$$\frac{1}{1+r}(A_{t+1} + C_{t+1} - Y_{t+1})$$

$A_t < 0$

PONZI SCHEME

$$A_{t} = \left(\frac{1}{1+r}\right)^T A_{t+T+1} + \frac{1}{1+r} \sum_{j=0}^{T} \left(\frac{1}{1+r}\right)^j (C_{t+j} - Y_{t+j})$$

We impose $\lim_{T \to +\infty} \left(\frac{1}{1+r}\right)^T A_{t+T+1} = 0 =$ TRANSVERSALITY CONDITION

(= condition for optimality)

with the T.C. and rewriting as of period t:

$$A_t = \frac{1}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \left(C_{t+j} - Y_{t+j}\right)$$

or $$A_t + \sum_{j=0}^{\infty} \frac{Y_{t+j}}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{C_{t+j}}{(1+r)^j}$$

- $C_{t+j} - Y_{t+j}$: can it be <u>always</u> > 0 ?

Human Wealth

$$H_t = \frac{1}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+n}\right)^j y_{t+j}$$

Assume $y_{t+j} = \bar{y} \quad \forall j$

Then $$H_t = \frac{1}{1+n} \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j \bar{y}$$

$$= \bar{y} \times \frac{1}{1+n} \times \sum_{j=0}^{\infty} \left(\frac{1}{1+n}\right)^j$$

$$H_t = \bar{y} \frac{1}{1+n} \times \left(\frac{1}{1-\frac{1}{1+n}}\right) = \bar{y} \frac{1}{1+n} \frac{1+r}{r} = \frac{\bar{y}}{r}$$

Undetermined coefficients

$C_t = \phi_1 A_t + \phi_2 \bar{y}$

- $A_{t+1} = A_t$ from Euler $= n A_t + \bar{y}$

- BC $A_{t+1} = (1+r) A_t + \bar{y} - C_t$

$A_{t+1} = A_t$

$C_t = \phi_1 A_t + \phi_2 \bar{y}$

$\Leftrightarrow \quad A_t = (1+r) A_t + \bar{y} - \phi_1 A_t - \phi_2 \bar{y}$

$\Leftrightarrow \quad (r - \phi_1) A_t + (1 - \phi_2) \bar{y} = 0 \quad \forall A_t, \forall \bar{y}$

$\Rightarrow \quad r - \phi_1 = 0, \quad 1 - \phi_2 = 0$

$\phi_1 = r, \quad \phi_2 = 1$

Rational Expectations

- Assume I need to forecast X_{t+1}
- I need to form $E_t X_{t+1}$
- Expectation error $\varepsilon^x_{t+1} = X_{t+1} - E_t X_{t+1}$

Rational expect $\Rightarrow$ ε^x has some remarkable properties

(1) $E_t \varepsilon^x_{t+1} = 0$

(2) $cov(\varepsilon^y_{t+1}, \varepsilon^x_t) = 0$

(1) $\varepsilon^{Y}_{t+1} = X_{t+1} - E_t X_{t+1}$

$E_t \varepsilon^{X}_{t+1} = E_t\left[X_{t+1} - E_t X_{t+1}\right]$

$= E_t X_{t+1} - E_t\left(E_t X_{t+1}\right)$

$= E_t X_{t+1} - E_t X_{t+1} = 0$

I am not systematically wrong.

(2) $Cov(\varepsilon_t^x, \varepsilon_{t+1}^x) = 0$

$$L> = E_t(X_t X_{t+1}) - E_t(X_t) E_t(X_{t+1})$$

$$= 0$$