FRANCK PORTIER
2016-2017

M1 MACRO
GROUP B
Session II

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

$$\max_{A_{t+1}, A_{t+2}, \ldots} E_t \Big[u\big[(1+r)A_t + Y_t - A_{t+1}\big] + \beta u\big[(1+r)A_{t+1} + Y_{t+1} - A_{t+2}\big] + \ldots \Big]$$

FOC wrt A_{t+1} : $E_t \Big[-u'(c_t) + \beta(1+r)\, u'(c_{t+1}) \Big] = 0$

EULER equation

$$\Leftrightarrow + E_t\big[u'(c_t)\big] = \beta(1+r)\, E_t\big[u'(c_{t+1})\big]$$

$$u'(c_t) = \underbrace{1}_{\text{(BY ASSUMPTION)}} E_t\big[u'(c_{t+1})\big]$$

STOCHASTIC EULER EQUATION:

$$u'(c_t) = \beta E_t\, u'(c_{t+1})$$

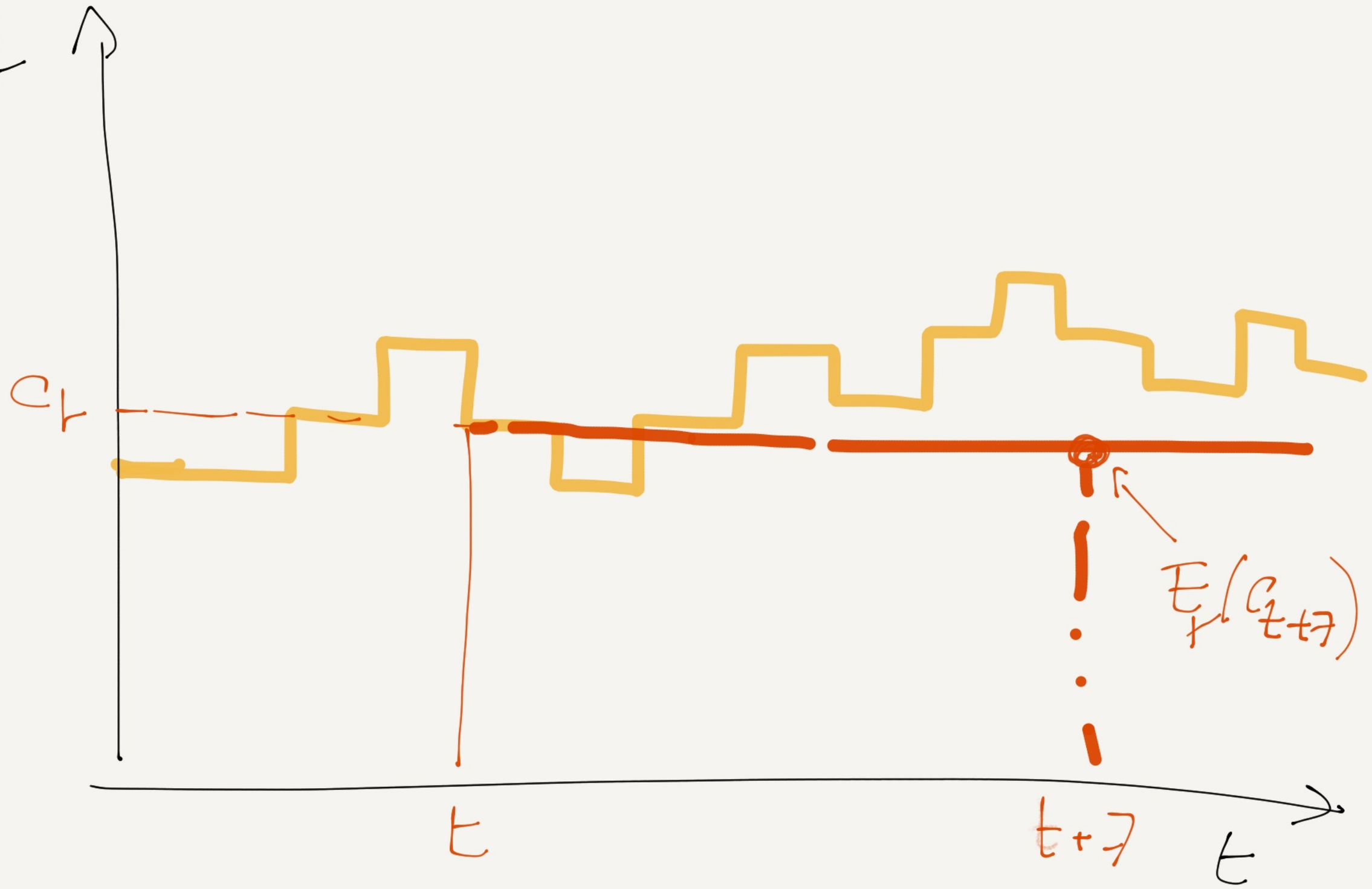
ASSUMPTION $\quad u(c_t) = \alpha_0 c_t - \frac{\alpha_1}{2} c_t^2$

$$u'(c_t) = \alpha_0 + \alpha_1 c_t$$

$$\alpha_0 + \alpha_1 c_t = \beta E_t(\alpha_0 + \alpha_1 c_{t+1})$$

$$c_t = E_t(c_{t+1})$$

CONSUMPTION FOLLOWS A RANDOM WALK



$$A_t = \frac{1}{1+r} A_{t+1} + Y_t - C_t$$

$$= \frac{1}{1+r}\left(\frac{1}{1+r} A_{t+2} + Y_{t+1} - C_{t+1}\right) + Y_t - C_t$$

$$= \left(\frac{1}{1+r}\right)^2 A_{t+2} + \frac{1}{1+r}\left(Y_{t+1} - C_{t+1}\right) + \left(Y_t - C_t\right)$$

$$\vdots \quad \vdots \quad \vdots$$

$$A_t = \left(\frac{1}{1+r}\right)^{T+1} A_{t+T+1} + \frac{1}{1+r} \sum_{i=0}^{T} \frac{Y_{t+i} - C_{t+i}}{(1+r)^i}$$

$$E_t A_t = E_t \left[\right.$$

$$\left. \right]$$

$$A_t = \lim_{T \to \infty} E_t \frac{A_{t+T+1}}{(1+r)^{T+1}} + \frac{1}{1+r} \sum_{i=0}^{\infty} \frac{E_t(Y_{t+i}) - E_t(G_{t+i})}{(1+r)^i}$$

$$= 0$$

TRANSVERSALITY CONDITION

$\Leftrightarrow$

WEALTH

$$A_t + \underbrace{\frac{1}{1+r} \sum_{i=0}^{\infty} \frac{E_t Y_{t+i}}{(1+r)^i}}_{H_t} = \frac{1}{1+r} \sum_{i=0}^{\infty} \frac{E_t(G_{t+i})}{(1+r)^i}$$

TOTAL (DISC.) EXPENDITURES

EULER eq.

$$\frac{1}{1+r}\left(C_{t+i}\right) = C_t \qquad \forall i$$

Therefore

$$A_t + H_t = \frac{1}{1+r}\sum \frac{C_t}{(1+r)^i}$$

$$= \frac{C_t}{1+r} \times \frac{1}{1-\frac{1}{1+r}} = \frac{C_t}{r}$$

Solution

↳

$$C_t = r\,(A_t + H_t)$$

SPECIFY A PROCESS FOR Y_t

$$H_t = \frac{1}{H2} \sum_{i=0}^{\infty} \frac{E_t Y_{t+i}}{(1+r)^i}$$

$$Y_t = \overline{Y} + \tilde{Y}_t$$

$\overline{Y}$ — permanent component of income

$\tilde{Y}_t$ — temporary component of income

Assume

$$\tilde{Y}_t = \rho \tilde{Y}_{t-1} + \varepsilon_t \quad , \quad 0 \leq \rho \leq 1$$

$\longrightarrow$ AR(1) process

Simple case $\boxed{\rho = 0}$: $\tilde{y}_t = \varepsilon_t$

$$\longrightarrow \underset{t}{E}\, \tilde{y}_{t+i} = \underset{t}{E}\, \varepsilon_{t+i} = 0$$

$$H_t = \frac{1}{1+r} \sum_{i=0}^{\infty} \frac{E_t(y + \tilde{y}_{t+i})}{(1+r)^i}$$

$$= \underbrace{\frac{1}{1+r} \sum_{i=0}^{\infty} \frac{y}{(1+r)^i}}_{\frac{y}{r}} + \frac{1}{1+r}\left[\underbrace{E_t \tilde{y}_t}_{\varepsilon_t} + \underbrace{\sum_{i=1}^{\infty} \frac{E_t \tilde{y}_{t+i}}{(1+r)^i}}_{0}\right]$$

$$= y/r + \varepsilon_t/(1+r)$$

Then,

$$C_t = \alpha A_t + \alpha\left(\frac{\bar{Y}}{\alpha} + \frac{\varepsilon_t}{1+\alpha}\right)$$

$$C_t = \alpha A_t + \bar{Y} + \frac{\alpha}{1+\alpha}\varepsilon_t$$

→ Komponen Temporer

→ Permanent Component

Marginal propensity to consume

- Permanent income $\quad \frac{dC_t}{d\bar{Y}} = 1$

- Temp income : $\quad \frac{dC_t}{d\varepsilon_t} = \frac{\alpha}{1+\alpha}$

Now look at the case $\tilde{y}_t = \rho \tilde{y}_{t-1} + \varepsilon_t$

↳ $E_t(\tilde{y}_{t+1}) = E_t(\rho \tilde{y}_t + \varepsilon_{t+1}) = \rho E_t(\tilde{y}_t) + E_t(\varepsilon_{t+1})$

$= \rho \tilde{y}_t$

$$\boxed{E_t[\tilde{y}_{t+i}] = \rho^i \tilde{y}_t}$$ ✓

Then $H_t = \frac{\bar{y}}{r} + \frac{1}{1+r} \sum_{i=0}^{\infty} \frac{E_t\, y_{t+i}}{(1+r)^i}$

$= \frac{\bar{y}}{r} + \frac{1}{1+r} \tilde{y}_t \times \sum_{i=0}^{\infty} \frac{\rho^i}{(1+r)^i} = \frac{\bar{y}}{r} + \frac{\tilde{y}_t}{1+r} \times \frac{1}{1-\frac{\rho}{1+r}}$

$$y_t = \frac{\bar{y}}{n} + \frac{1}{1+n}\tilde{y}_t \quad \frac{+n}{1+n-\rho}$$

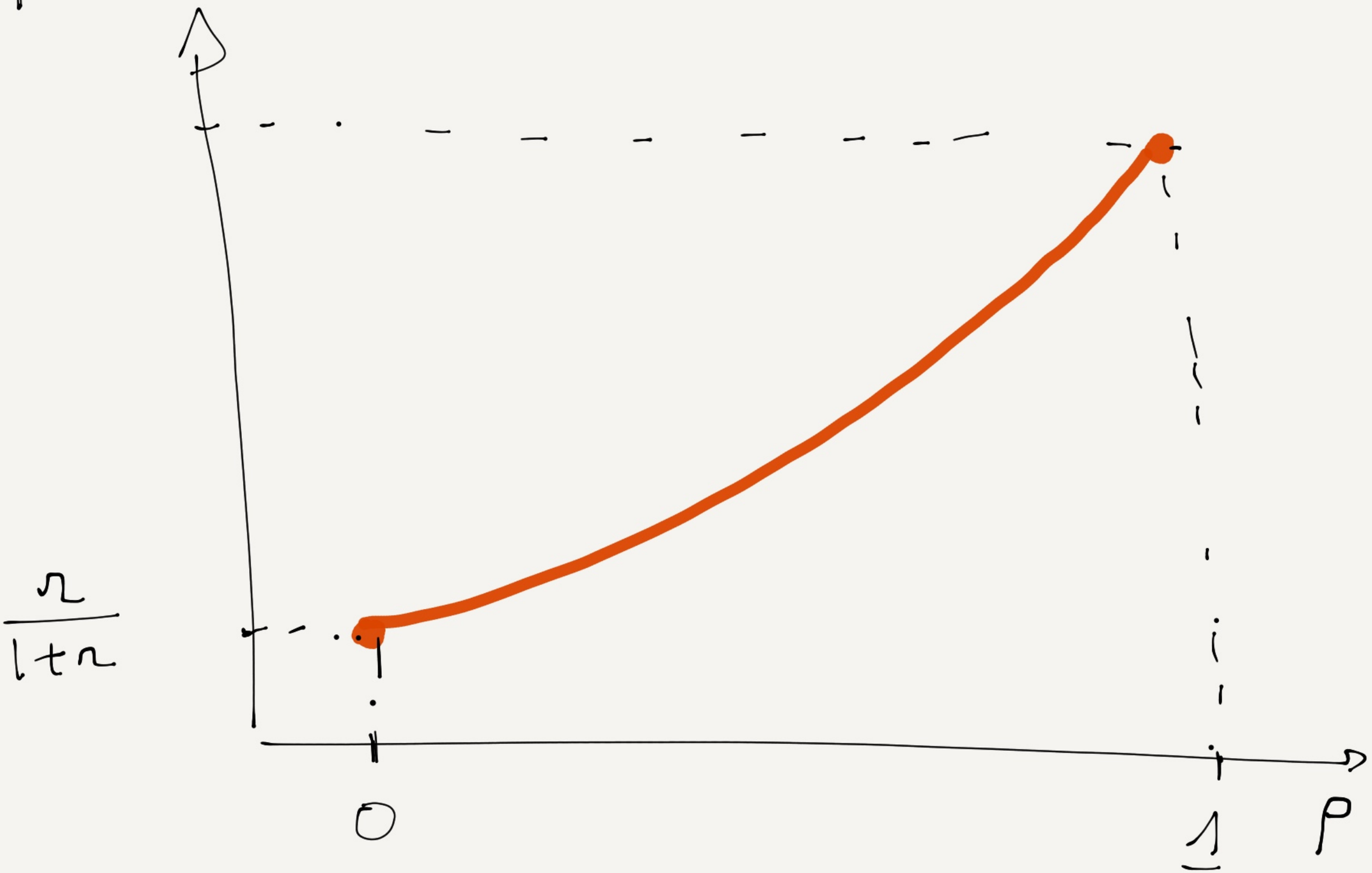
$$= \frac{\bar{y}}{n} + \frac{\tilde{y}_t}{1+n-\rho}$$

$$\hookrightarrow C_t = \bar{n} A_t + \underbrace{\bar{y}}_{\text{permanent}} + \frac{n}{1+n-\rho}\underbrace{\tilde{y}_t}_{\text{temporary but persistent}}$$

$$\frac{dC_t}{d\bar{y}} = 1, \quad \frac{dC_t}{d\tilde{y}_t} = \frac{n}{1+n-\rho}$$

$$\frac{1}{-r}$$

mpc



Note $\quad E f(X) \neq f(E(X))$

$\left(= \text{if } f \text{ is linear} \right)$