FRANCK PORTIER
2016-2017

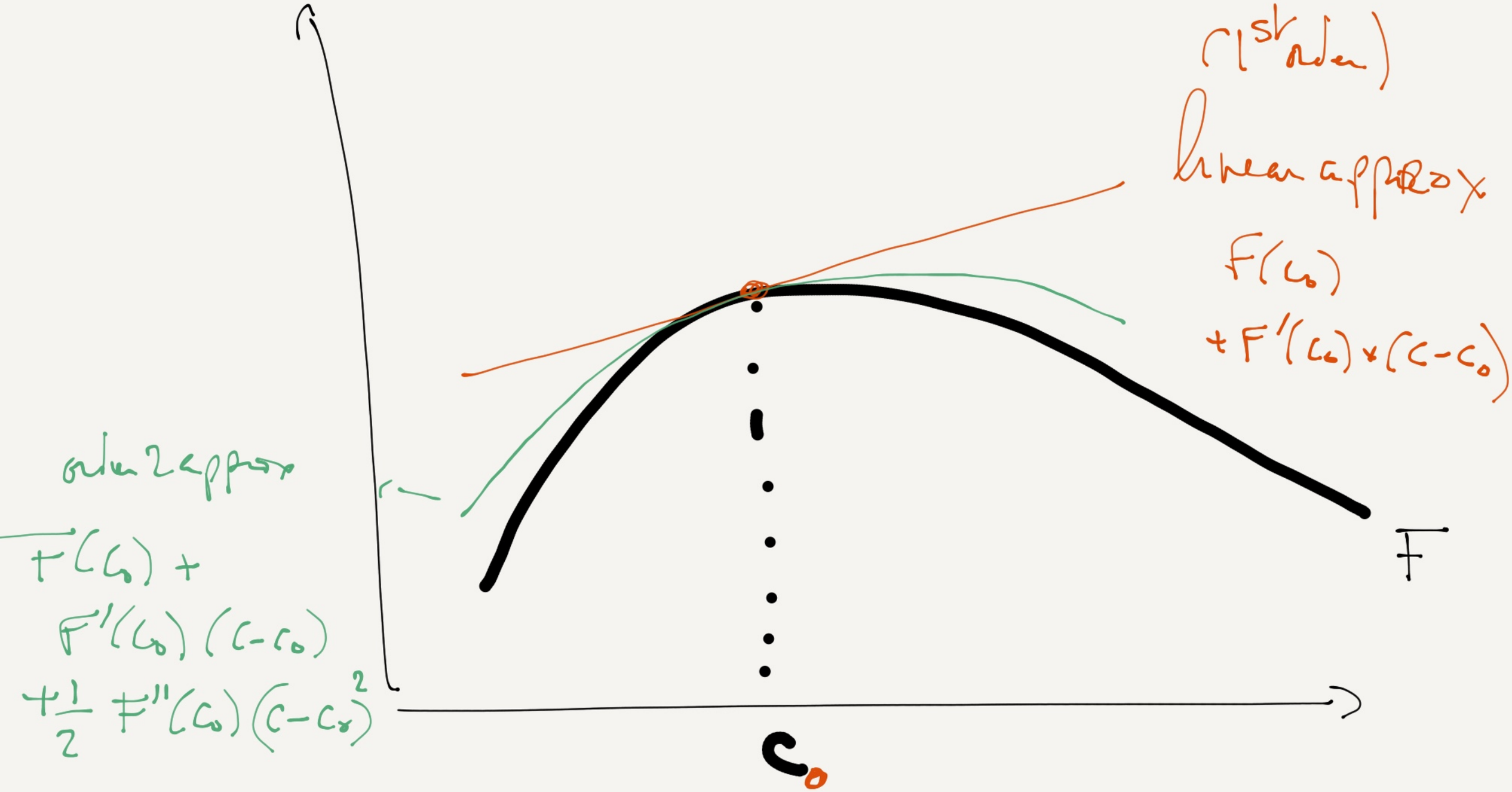
M1 MACRO

GROUP B

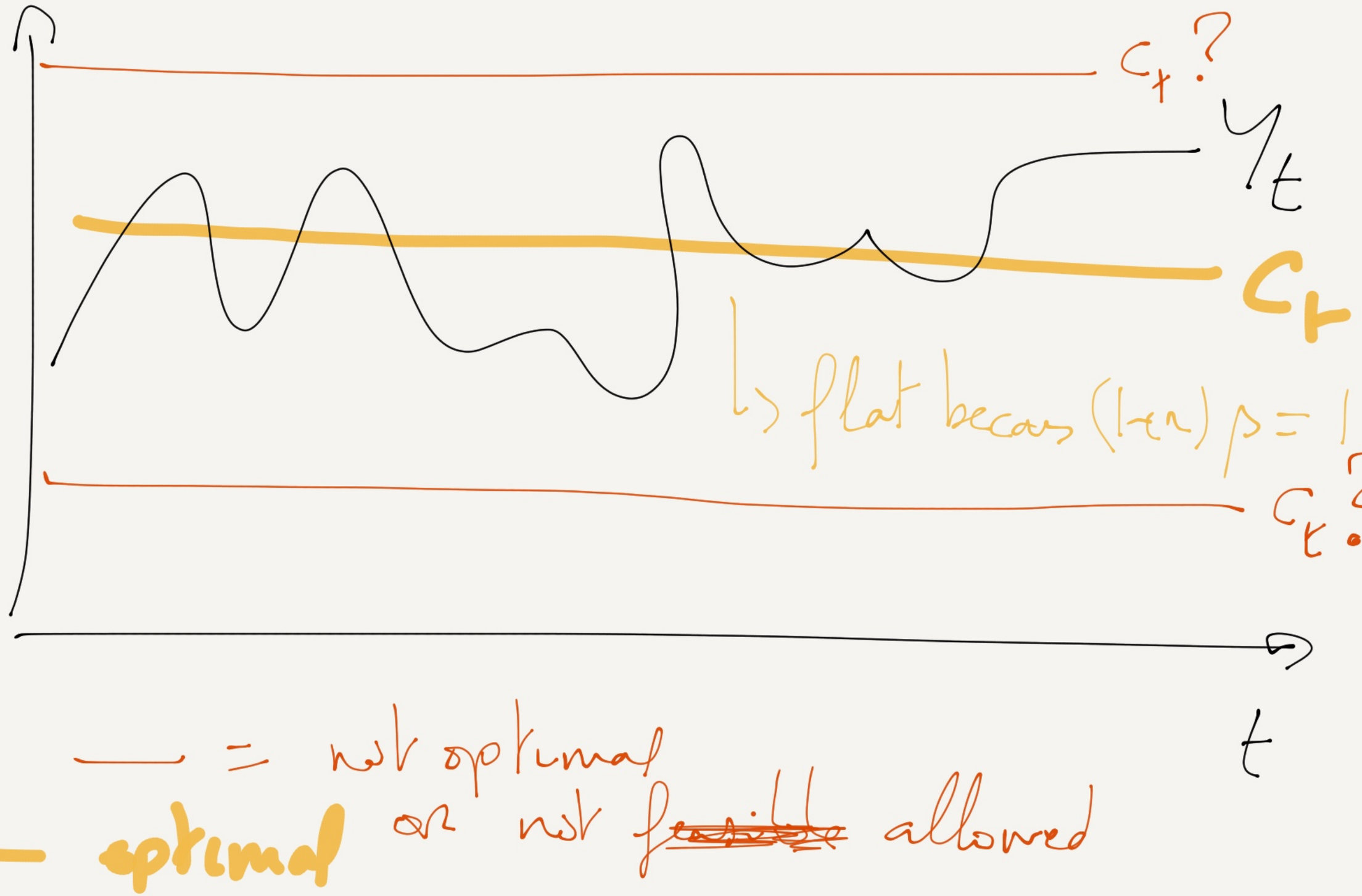
Session 8

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

TAYLOR EXPANSIONS



Deterministic model



—— = not optimal or not ~~feasible~~ allowed

— optimal

$$A_{t+1} = (1+r)A_t - C_t + Y_t$$

$$\Leftrightarrow A_t = \frac{1}{1+r} A_{t+1} - \frac{Y_t}{1+r} + \frac{C_t}{1+r}$$

In other time is $t+1$: $A_{t+1} = \frac{1}{1+r} A_{t+2} - \frac{Y_{t+1}}{1+r}$

$$\rightarrow A_t = \frac{1}{1+r}\left[\frac{1}{1+r} A_{t+2} - \frac{Y_{t+1}}{1+r} + \frac{C_{t+1}}{1+r} - Y_t + C_t\right] + \frac{C_{t+1}}{1+r}$$

$$\hookrightarrow A_{t+2} = \frac{1}{1+r} A_{t+3} - \frac{Y_{t+2}}{1+r} + \frac{C_{t+2}}{1+r}$$

$$\Rightarrow A_t = \left(\frac{1}{1+r}\right)^T A_{t+T} + \frac{1}{1+r} \sum_{j=0}^{T-1} \left(\frac{1}{1+r}\right)^j (C_{t+j} - Y_{t+j})$$

$$A_t = \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T A_{t+T} + \lim_{T\to\infty} \sum_{j=0}^{T-1} \left(\frac{1}{1+r}\right)^{T-1} (c_{t+j} - y_{t+j})$$

- not optimal to be > 0
- not allowed to be < 0

Hence $\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T A_{t+T} = 0$

$=$ TRANSVERSALITY CONDITION (TC)

(Intuition: Assume that the world ends at $t+T-1$)

$$A_t + H_t = \frac{1}{1+r} \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j C_{t+j}$$

$C_t + A_{t-1}(1+r) = (1+r) A_t + Y_t$

From the Euler eq., $C_{t+j} = C \;\forall j$

$$\rightarrow \quad A_t + H_t = \frac{C}{1+r} \times \underbrace{\sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j}_{\frac{1}{1 - \frac{1}{1+r}}} \qquad (r > 0)$$

$$A_t + H_t = \frac{C}{r} \quad \Rightarrow C = \boxed{C = r(A_t + H_t)}$$