FRANCK PORTIER
2016-2017

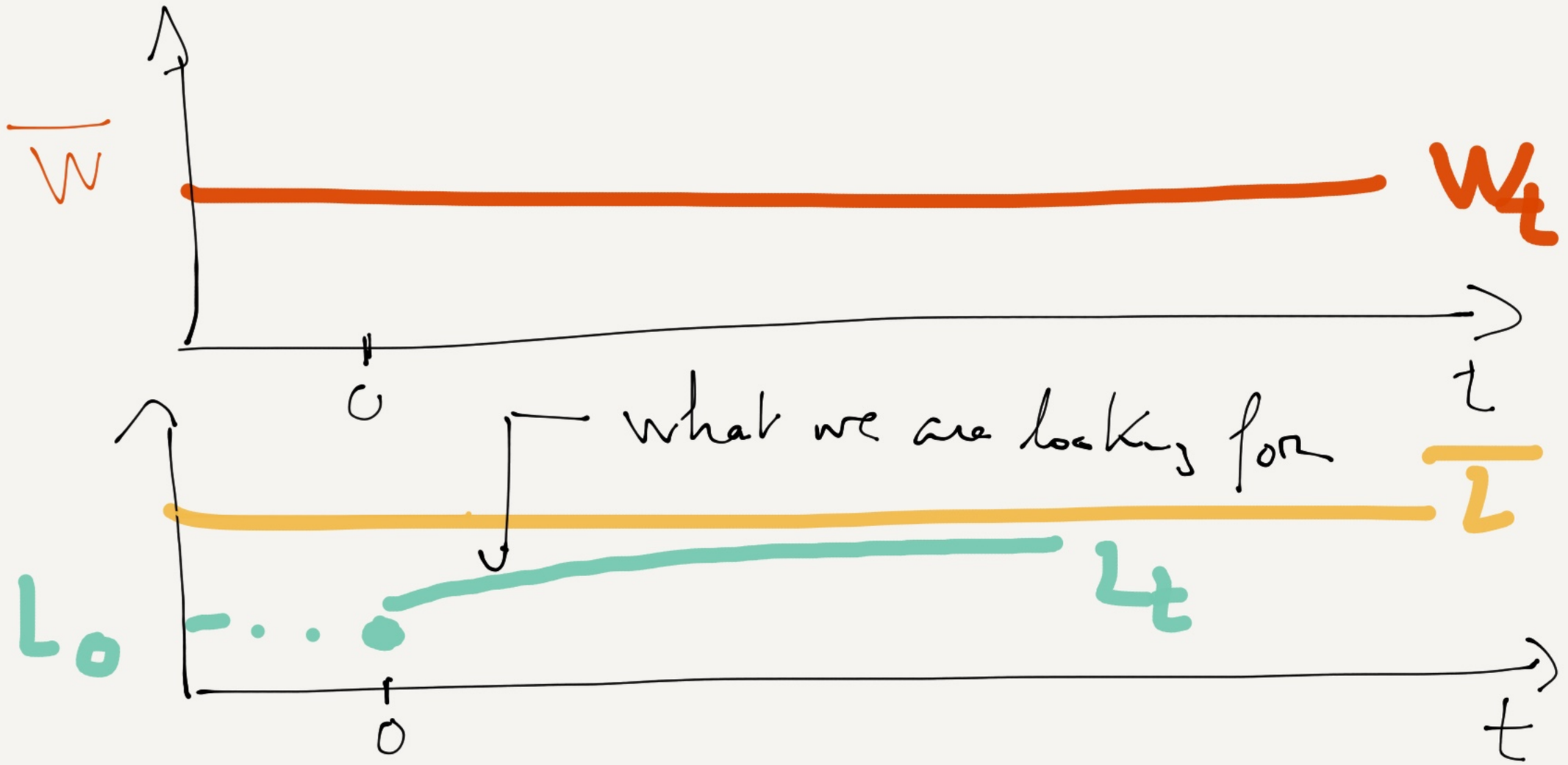
M1 MACRO
GROUP B
Session 14

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

$$\beta b\,\widehat{\Sigma}_{t+1} - (d_1 + b + \beta b)\widehat{\Sigma}_t + b\,\widehat{\Sigma}_{t-1} = 0$$

[Assume $\widehat{w}_t = 0$]

Note $\widehat{w}_t = 0 \;\forall t \Rightarrow w_t = \overline{w}$



How to find c_1 and c_2?

- $\Sigma_0 = c_1 \lambda_1^0 + c_2 \lambda_2^0 = c_1 + c_2$ ← 1 eq in (c_1, c_2)

  Known

- Transversality condition: $\lim_{t \to \infty} \Sigma_t = 0$

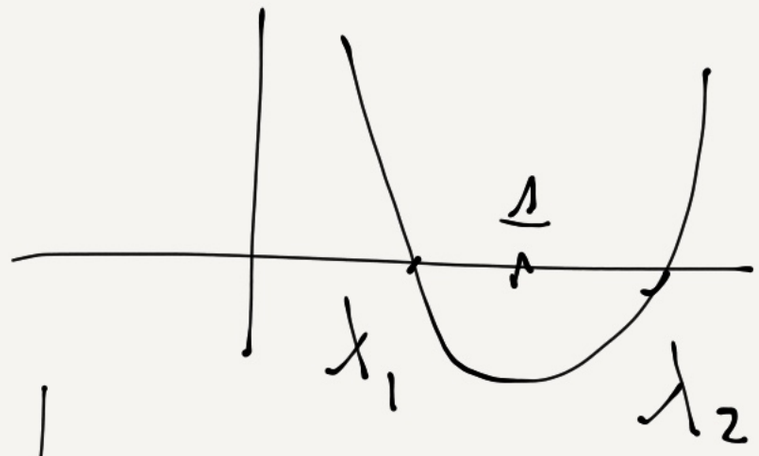
  $\Leftrightarrow \lim_{t \to \infty} L_t = \bar{L}$

  ↳ $\lim_{t \to \infty} c_1 \lambda_1^t + c_2 \lambda_2^t = c_1 \lim_{t \to \infty} \lambda_1^t + \lim_{t \to \infty} c_2 \lambda_2^t = 0$

  $\lim_{t \to \infty} \lambda_1^t = 0$ if and only if $c_2 = 0$

I know that the solution is

$$L_t = c_1 \lambda_1^t + c_2 \lambda_2^t$$



where c_1, c_2 have to be found

- λ_1, λ_2 are the roots of $\beta b \lambda^2 - (\alpha_1 + b + \beta b)\lambda + b = 0$
- We have shown that $0 < \lambda_1 < 1$
  $\lambda_2 > 1$
- L_0 is given

$$\Longrightarrow \quad \lambda_{1,2} = \frac{(\alpha_1 + b + \beta b) \pm \sqrt{(\alpha_1 + b + \beta b)^2 - 4\beta b^2}}{2\beta b}$$

$$\Rightarrow c_2 = 0, \quad c_1 = \hat{L}_0 = L_0 - \bar{L}$$

therefore, the optimal path is $\hat{L}_t = \hat{L}_0 \lambda_1^t$

$$= \lambda_1 \times \underbrace{\hat{L}_0 \lambda_1^{t-1}}_{\hat{L}_{t-1}}$$

$$\Rightarrow \boxed{\hat{L}_t = \lambda_1 \hat{L}_{t-1}}$$

$$L_t - \bar{L} = \lambda_1 (L_{t-1} - \bar{L}) \qquad \bar{L} = \frac{d_0 - \bar{w}}{\alpha_1}$$

$$\Leftrightarrow \boxed{L_t = (1-\lambda_1)\bar{L} + \lambda_1 L_{t-1}} \; ; L_0 \text{ given}$$

Description of the optimal path

- $w_t = \bar{w}$ $\forall$ t from $-\infty$ to $+\infty$

- Assume L_0 is given

$$L_1 = (1-\lambda_1)\bar{L} + \lambda_1 L_0 = \bar{L} - \lambda_1(\bar{L} - L_0)$$

$\Rightarrow$ λ_1 is here a decreasing function of b

high b $\Rightarrow$ high adj. costs $\Rightarrow$ slow adjustment

$\Leftarrow$ λ_1 is close to $\underline{1}$ $\Rightarrow$ L_1 close to L_0

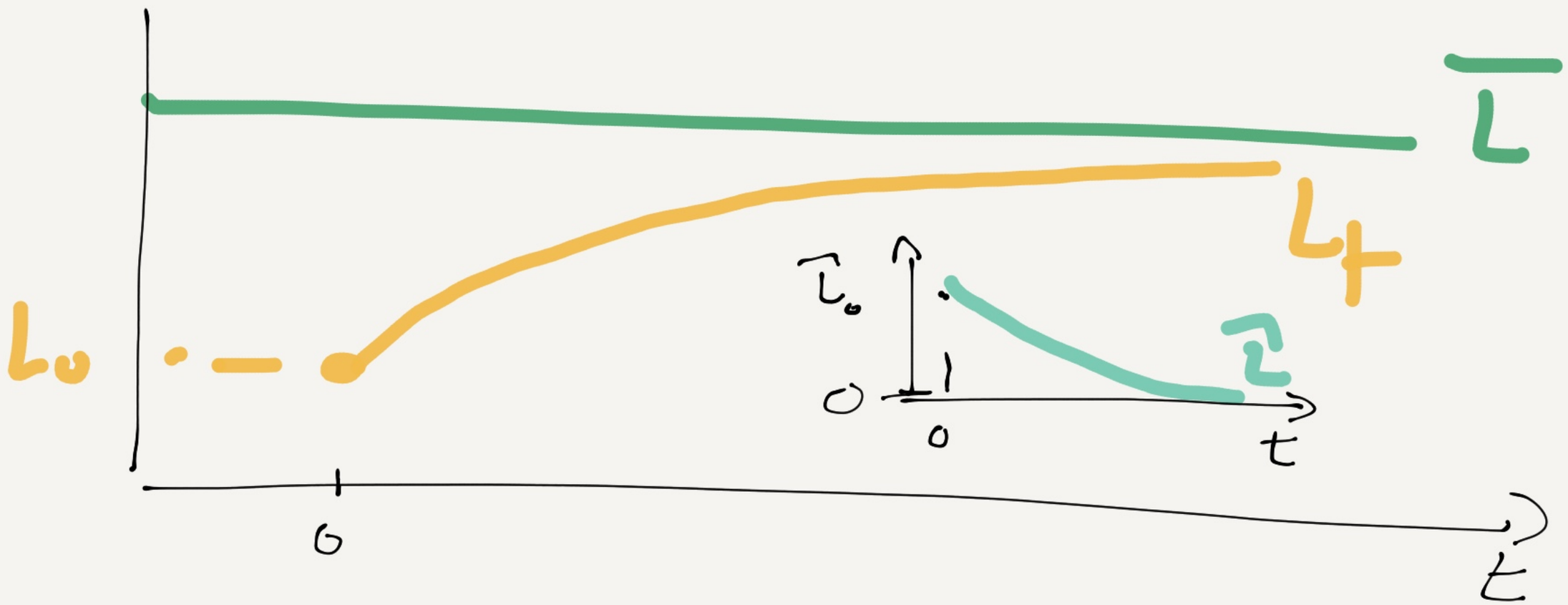
$$L_2 = (1-\lambda_1)\bar{L} + \lambda_1 L_1 = (1-\lambda_1)(1+\lambda_1)\bar{L} + \lambda_1^2 L_0$$

$$L_3 = (1-\lambda_1)\bar{L} + \lambda_1 L_2 = (1-\lambda_1)(1+\lambda_1+\lambda_1^2)\bar{L} + \lambda_1^3 L_0$$

$$L_t = \overline{L}\left(1-\lambda_1\right)\sum_{j=0}^{t-1}\lambda_1^j + \lambda_1^t L_0$$

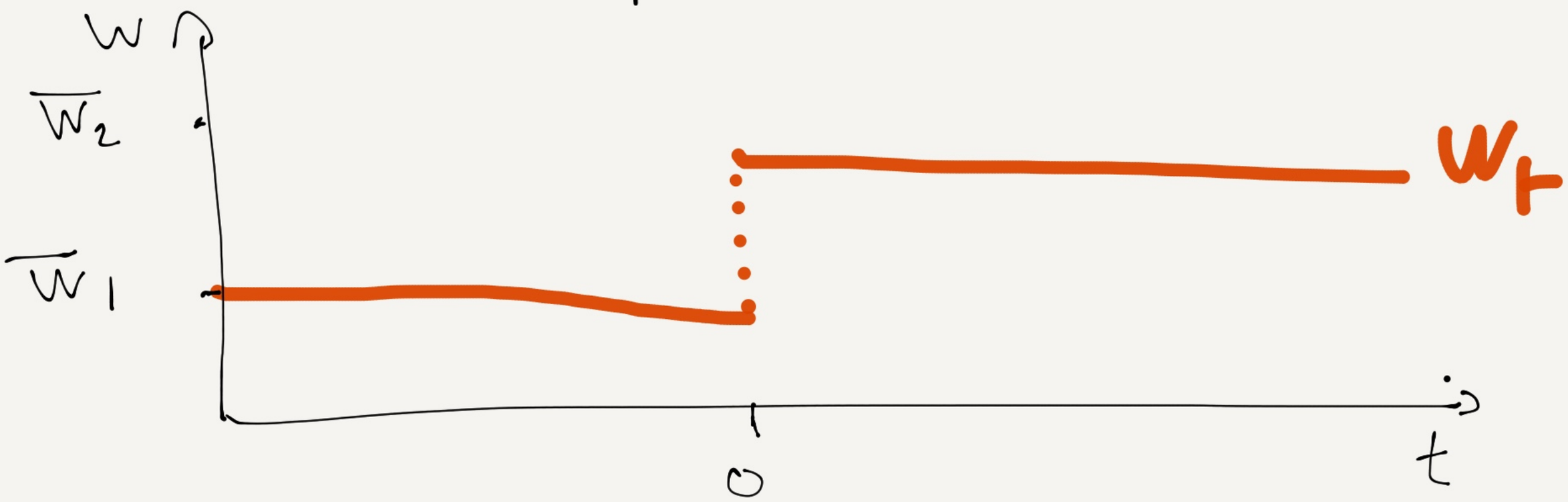
$$\lim_{t\to+\infty} L_t = \left(1-\lambda_1\right)\overline{L}\,\frac{1}{1-\lambda_1} + 0$$

$$= \overline{L}$$

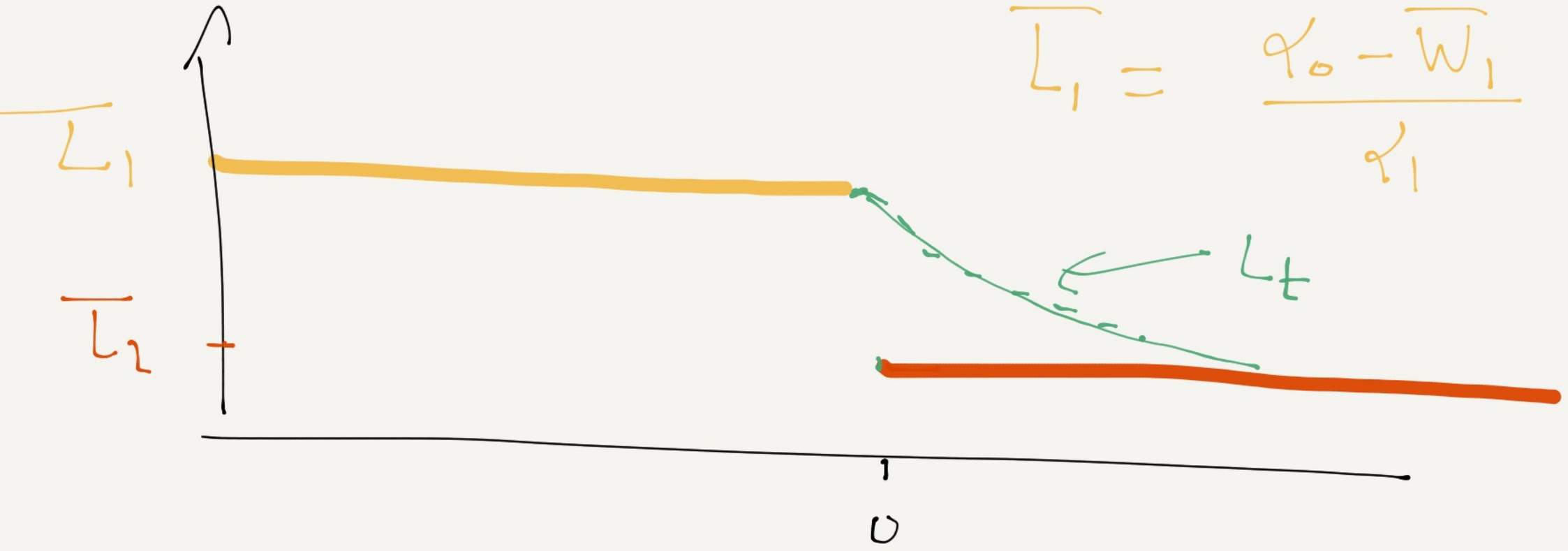


Experiment

- from $-\infty$ to -1, $W_t = \overline{W_1}$
- W_t is expected to stay at $\overline{W_1}$ for ever
- Unexpectedly, in 0, $W_0 = \overline{W_2} > \overline{W_1}$
- this will last for ever.



- Assume $L_t = \overline{L}_1$ for $t < 0$



$$\overline{L}_1 = \frac{q_0 - \overline{w}_1}{q_1}$$

$$\overline{L}_2 = \frac{q_0 - \overline{w}_2}{q_1} < \overline{L}_1$$

$$L_0 = (1-\lambda_1)\overline{L_2} + \lambda_1 L_{-1} = (1-\lambda_1)\overline{L_2} + \lambda_1 \overline{L_1}$$

$$L_1 = (1-\lambda_1)\overline{L_2} + \lambda_1 L_0$$

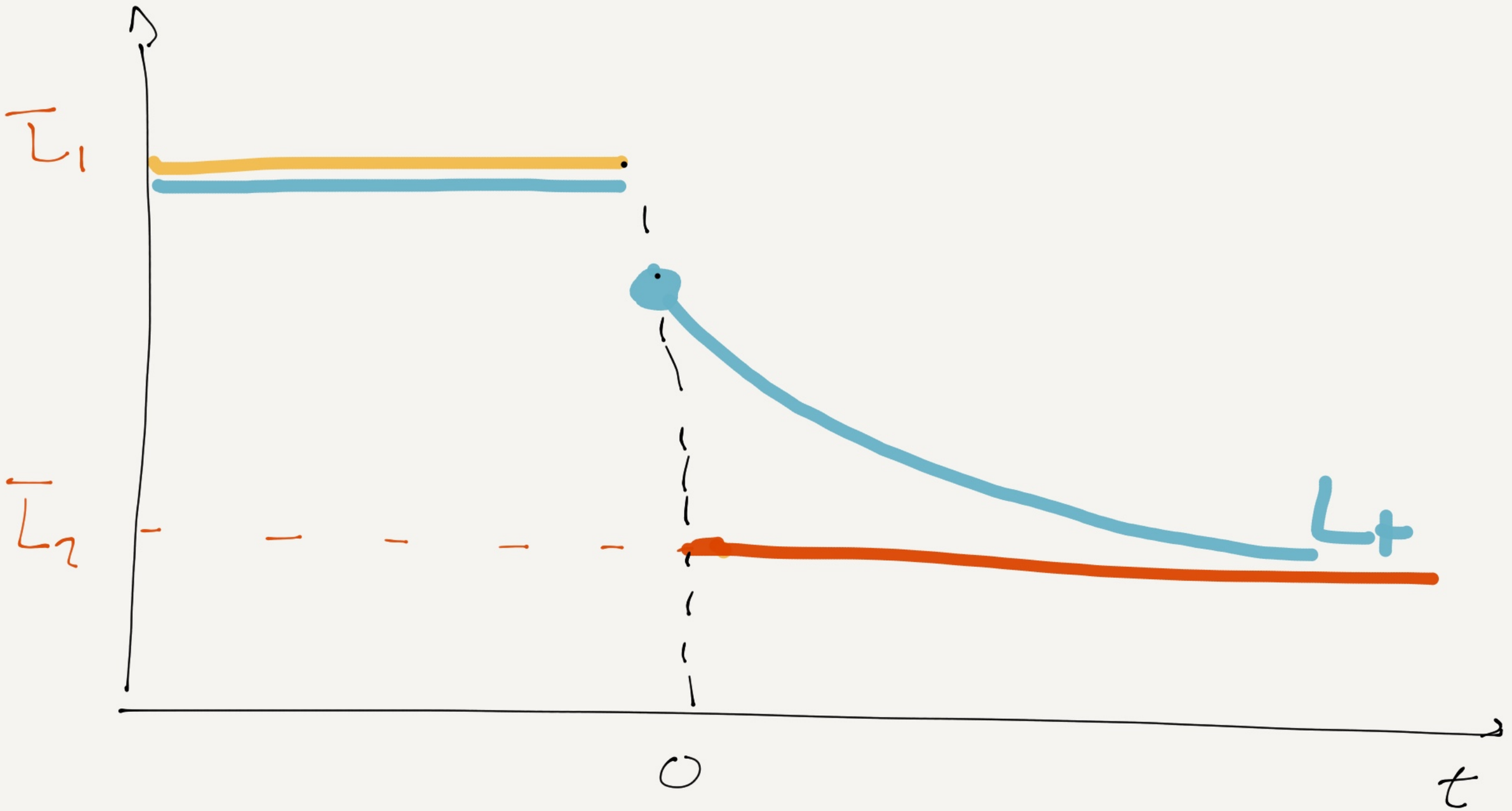
$$= (1-\lambda_1)\overline{L_2} + \lambda_1(1-\lambda_1)\overline{L_2} + \lambda_1^2 \overline{L_1}$$

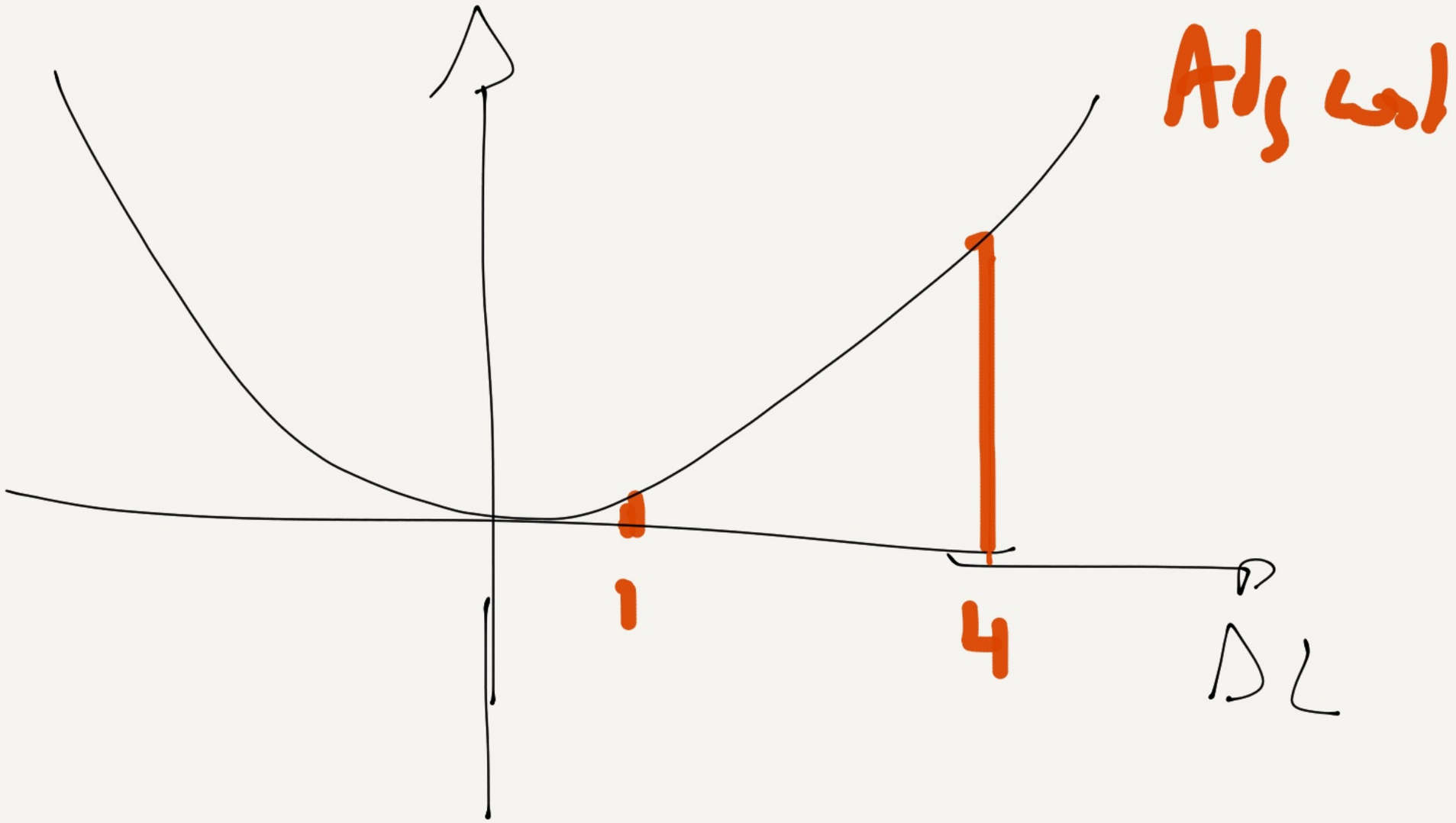
$$= (1-\lambda_1)(1+\lambda_1)\overline{L_2} + \lambda_1^2 \overline{L_1}$$

$$\vdots$$

$$L_t = \overline{L_2}(1-\lambda_1)\sum_{j=0}^{t-1} \lambda_1^j + \lambda_1^t \overline{L_1}$$

$$\lim_{t \to \infty} L_t = \overline{L_2}$$


$\overline{L}_1$
$\overline{L}_2$
L_+
0
t


1
4