FRANCK PORTIER
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M1 MACRO

GROUP B

Session 15

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

$$\begin{pmatrix} L_{t+1} \\ L_1 \end{pmatrix} = A \begin{pmatrix} L_t \\ L_{1:-1} \end{pmatrix} = A^t \begin{pmatrix} L_1 \\ L_0 \end{pmatrix}$$

NOT given

given

$x_{t:1}$ $\quad$ x_t

A is "well behaved":

$\exists P$ st $PAP^{-1} = \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

where λ_1, λ_2 are the 2 eigenvalues of A

Result : $a\lambda^2 + b\lambda + c = 0$ $\quad \lambda_1, \lambda_2 \neq 0$

$\rightarrow$ roots λ_1, λ_2

then $\quad a + b\mu + c\mu^2 = 0$

$\rightarrow$ roots : $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}$

Proof

$a + b\mu + c\mu^2 = 0$

$(\mu \neq 0)$ change of variable : $\lambda = \frac{1}{\mu} \Rightarrow a + \frac{b}{\lambda} + \frac{c}{\lambda^2} = 0$

$\Rightarrow \lambda^2 [a\lambda^2 + b\lambda + c] = 0 \Rightarrow$ roots : λ_1, λ_2 .

$$0 = -\lambda^2 + b\lambda + c \iff a(\lambda - \lambda_1)(\lambda - \lambda_2) = 0$$

$$0 = a + b\lambda + c\lambda^2 \iff c\left(\lambda - \frac{1}{\lambda_1}\right)\left(\lambda - \frac{1}{\lambda_2}\right) = 0$$

$$\iff = c_1 \lambda_1 \lambda_2 \left(\lambda\lambda_1 - 1\right)\left(\lambda\lambda_2 - 1\right) = 0$$

$$\iff 0 = (t t_1 - 1)(t t_2 - 1)$$

$$\tilde{X}_{t+1} = A \tilde{X}_t$$

$$\Leftrightarrow P^{-1} \tilde{X}_{t+1} = P^{-1} A \tilde{X}_t$$

$$\underbrace{P^{-1} \tilde{X}_{t+1}}_{} = \underbrace{P^{-1} A P}\underbrace{P^{-1} \tilde{X}_t}$$

$$Z_{t+1} = \Lambda \quad Z_t$$

$$\Leftrightarrow \begin{bmatrix} z^1_{t+1} \\ z^2_{t+1} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} z^1_t \\ z^2_t \end{bmatrix} = \begin{bmatrix} \lambda_1^t & 0 \\ 0 & \lambda_2^t \end{bmatrix} \begin{bmatrix} z^1_0 \\ z^2_0 \end{bmatrix}$$

A

$\det(A - \lambda I) = 0$

2 solutions: λ_1, λ_2

eigenvalues

$\lambda_1 \rightsquigarrow v_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$A v_1 = \lambda_1 v_1$

$$Z = P^{-1} X \iff X = P Z$$

$$\iff \begin{bmatrix} \Gamma_t \\ \Gamma_{t-1} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} Z_t^1 \\ Z_t^2 \end{bmatrix}$$

In period 1:

$$\begin{cases} \Gamma_1 = P_{11} Z_1^1 + P_{12} Z_1^2 \\ \Gamma_0 = P_{21} Z_0^1 + P_{22} Z_0^2 \end{cases}$$

What is known?

$P_{11}, P_{12}, P_{21}, P_{22}$

Γ_0.

$Z_0^2 = 0$ (Transv. Condition)

→ I can find Z_0^1, Γ_1