FRANCK PORTIER
2016-2017

M1 MACRO

GROUP B

Session 17

These notes are written during class. They are posted "for the record" and may contain mistakes; they are not intented to be read by anyone that did not attend the class.

Habit persistence GE model

Firms → Representative firm [competitive]

Max $\{y_t, h_t\}$ $\Pi_t = P_t Y_t - W_t h_t$

s.t. $Y_t = Z_t h_t$ ← productivity (exog)

$\Leftrightarrow$ Max $\{h_t\}$ $P_t Z_t h_t - W_t h_t = (P_t Z_t - W_t) h_t$

FOC: $\boxed{P_t Z_t = W_t}$ ← badly written

π_-

$p_t z_t > w_t$

$\longrightarrow h_t^d = +\infty$

$p_t z_t = w_t$ $\longrightarrow$ $h_t \in \mathbb{R}_+$

h_{t-}

$p_t z_t < w_t$

$\longrightarrow h_t^d = 0$

$\left[\text{if } \gamma_t = z_t h_{t-}^{\alpha} \right]_{0<\alpha<1}$

$$h_t^d(?) = \begin{cases} +\infty & \text{if } P_t Z_t > w_t \\ \in \mathbb{R}^+ & \text{if } P_t Z_t = w_t \\ 0 & \text{if } P_t Z_t < w_t \end{cases}$$

and $Y_t = Z_t h_t^d$

Hh Representative (competitive)

$$U = \sum_{0} \beta^t \left[\log(C_t - b C_{t-1}) - h_t \right]$$

U

h_t: disutility of labor

$0 \le b < 1$

= habit persistence

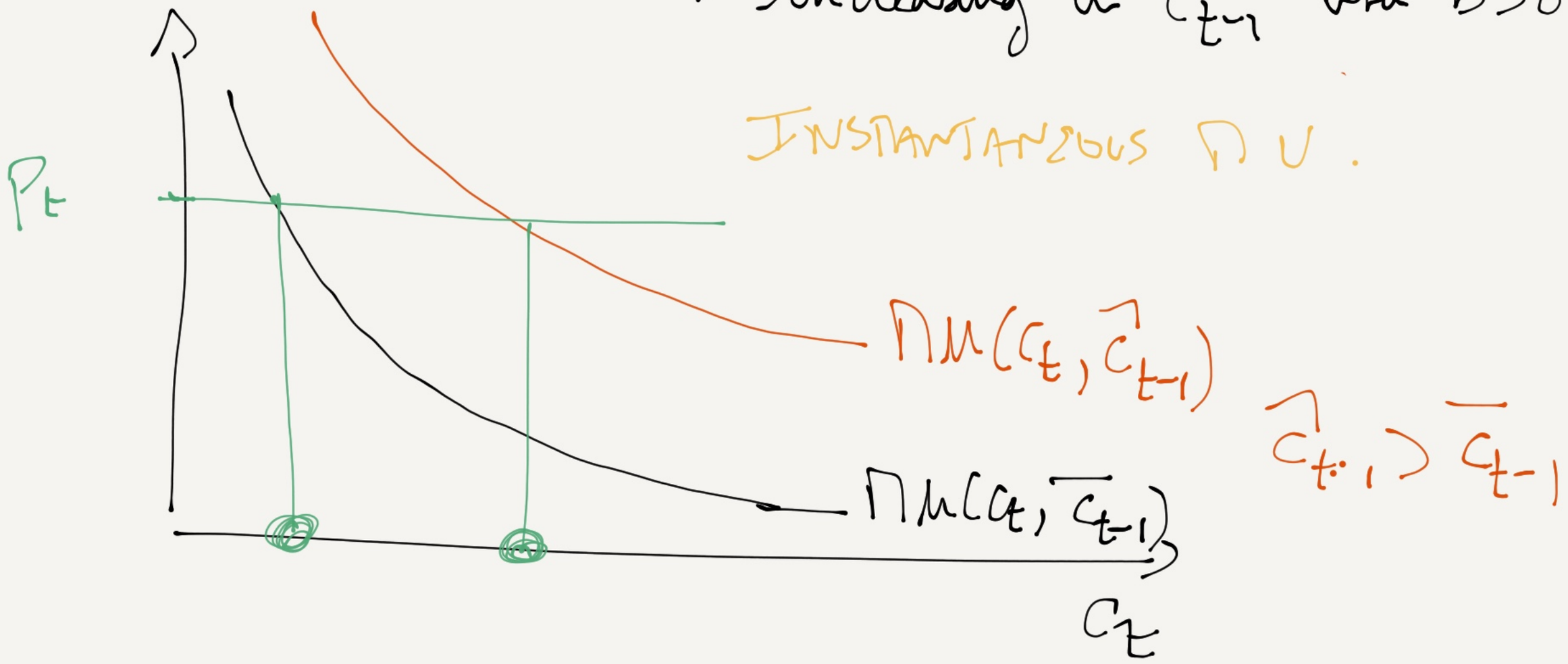
[if $b=0$, back to basic model]

$$mu\ \text{of}\ C_t: \quad \frac{dU}{dC_t} = \frac{\beta^t}{C_t - bC_{t-1}}$$

$b > 0 \rightarrow$ decreasing in C_t

$\rightarrow$ increasing in C_{t-1} when $b>0$

INSTANTANEOUS $\cap U$.



$\vec{C}_{t-1} > \bar{C}_{t-1}$

Budget constraint $P_t C_t = w_t h_t + \Pi_t, \forall t$

[No saving]

$$C_t = \frac{w_t h_t + \Pi_t}{P_t} \longrightarrow \text{plug in } U.$$

$$U = E_0 \sum \beta^t \left(\log\left(\frac{w_t h_t + \Pi_t}{P_t} - b \frac{(w_{t-1} h_{t-1} + \Pi_{t-1})}{P_{t-1}} \right) - h_t \right)$$

$\max_{\{h_t\}} U$

FOC wrt h_t

FOC wrt h_t in period t

$$\beta^t \left(\frac{w_t}{P_t} \times \frac{1}{C_t - b C_{t-1}} - \frac{1}{} \right) - \beta^{t+1} \frac{b \frac{w_t}{P_t}}{C_{t+1} - b C_t} = 0$$

t

$$\Leftrightarrow \quad \frac{1}{C_t - b C_{t-1}} - E_t \frac{\beta b}{C_{t+1} - b C_t} = \frac{1}{w_t / P_t}$$

C_0 given

+ transversality condition

G.E : A competitive GE is

(C_t, y_t, h_t^s, h_t^d) and (P_t, W_t) s.t

- for given prices, firms and HH max. objectives
- markets clear : $\begin{cases} C_t = y_t \\ h_t^s = h_t^d \end{cases}$

Note . Walras' law : $P_t = 1$ [numéraire]

. labor market eq $\Rightarrow$ $P_t Z_t = W_t$

$\Leftrightarrow$ $W_t = Z_t$

Euler eq. at equilibrium

$$\frac{1}{c_t - b c_{t-1}} - \beta b E_t \frac{1}{c_{t+1} - b c_t} = \frac{1}{z_t} \qquad (a)$$

Change of variable : $X_t = \frac{1}{c_t - b c_{t-1}}$

(a) $\Leftrightarrow$ $X_t - \beta b E_t X_{t+1} = \frac{1}{z_t}$

$0 < \quad < 1$

Solving forward

$$X_t = \frac{1}{Z_t} + \beta b E_t X_{t+1}$$

$$= \frac{1}{Z_t} + \beta b E_t \left[\frac{1}{Z_{t+1}} + \beta b E_{t+1} X_{t+2} \right]$$

$$= \frac{1}{Z_t} + \beta b E_t \left(\frac{1}{Z_{t+1}} \right) + (\beta b)^2 E_t X_{t+2}$$

$$\therefore X_t = \sum_{j=0}^{\infty} (\beta b)^j E_t \left(\frac{1}{Z_{t+j}} \right) + \underbrace{\lim_{j \to \infty} (\beta b)^j E_t X_{t+j}}_{= 0 \text{ (Transversality)}}$$