2017-2018 – DEEQA – TSE – Advanced Macroeconomics

Lecture 5 : Dynamics with Interactions

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0. Introduction

- Here I present first abstract static and dynamic models with strategic interactions
- These models have a macro flavour : a large number of atomistic agents
- I discuss the role of complementarities in generating multiplicity, multipliers, rich dynamics
- I then present an extended NK model that features rich dynamics (up to limit cycles)
- I show that it can produce boom-bust cycles and peaks in spectral density.

1. Static model - Cooper & John, 1988

Setup

- N players with payoffs function $V(e_j, e)$, where $e = \frac{1}{N}\sum_{j=1}^{N} e_j$
- $e \in [0, E]$
- $V_{11}(e_j, e) < 0$
- $e_j^\star(e)$ is the best response function, implicitly defined from

$$V_1(e_j, e) = 0;$$

- SCE is simply the planner solution

$$\max_e NV(e, e)$$

- FOC is

$$V_1(e, e) + V_2(e, e) = 0$$

- SOC is

$$V_{11}(e, e) + 2V_{12}(e, e) + V_{22}(e, e) < 0$$

- Assume for existence of an interior SCE

$$\lim_{e \to 0} V_1(e, e) + \lim_{e \to 0} V_2(e, e) > 0$$

$$\lim_{e \to E} V_1(e, e) + \lim_{e \to E} V_2(e, e) < 0$$

1. Static model - Cooper & John, 1988

Some definitions

- *Positive spillovers* : $V_2(e_j, e) > 0$
- *Negative spillovers* : $V_2(e_j, e) < 0$
- *Strategic complementarity* : $V_{12}(e_j, e) > 0$
- *Strategic subtitutability* : $V_{12}(e_j, e) < 0$
- *Multiplier* :
  - × $\frac{\partial e_j^\star(e;\theta)}{\partial \theta} = \frac{V_{1\theta}}{V_{11}}$ is the partial equilibrium response of e to θ
  - × $\frac{\partial e}{\partial \theta} = \frac{V_{1\theta}}{V_{11}+V_{12}}$ is the general equilibrium response of e to θ
  - × There are multipliers when the general equilibrium response is in the same direction as the partial equilibrium one, and greater.
  - × An example of no multipliers would be the consumption of apples when taste for apple increases in a walrasian equilibrium.

- Agent aims at $\max_{e_j} V(e_j, e)$
- Agent j takes optimal action *given* the actions of others.
- Either she takes into account her effect on e or she does not
  - × If she does, FOC is

$$V_1(e_j, e) + V_2(e_j, e)\frac{\partial e}{\partial e_j} = V_1(e_j, e) + \frac{1}{N}V_2(e_j, e) = 0$$

  - × If she does not

$$V_1(e_j, e) = 0$$

  - × I focus on the second case (simple, but does not affect results
  - × N large $\rightsquigarrow$ close to the second case, which is the maximum departure from social optimum, as we will see

- Given by $V_1(e_j, e) = 0$
- Slope is (by full differentiation)

$$\rho = \frac{de_j}{de} = -\frac{V_{12}}{V_{11}}$$

- Best response can be increasing, decreasing, non monotonous, non linear,...

- SNE defined by

$$V_1(e, e) = 0$$

- to be compared to the SCE

$$V_1(e, e) + V_2(e, e) = 0$$

- Prop 1 : if there are spillovers, then the SNE is inefficient
- Prop 2 : If spillovers are positive, then the SNE levels of e are two low
- Prop 3 : Strategic complementarities are necessary and sufficient for multipliers
- Note : When there are positive spillovers and strategic compl., may want to refer to such equilibrium outcome as exhibiting *"deficient demand"*

- Prop 4 : There can be multiple equilibria. Strategic complementarities (and $-\frac{V_{12}}{V_{11}} > 1$ at some point) are necessary for multiplicity
- Prop 5 : If positive spillovers and multiple equilibria, then equilibria are Pareto ranked with higher actions being better.
- Corollary : If $e_j^\star(e) = e$ over an interval, and there are positive spillovers, then there is a continuum of equilibrium increasing in e.

- A Walrasian equilibrium can be seen as a special case of the above game situation.
- In this case, we can see agents as choosing actions – expenditures or supply decisions – taking prices as given, but where prices depend on aggregate actions
- What differentiates the Walrasian equilibrium is at an equilibrium $V_2 = 0$, ie, there are no spillovers.
- Therefore, the Walrasian equilibrium is efficient
- This is not because agents are small, but because the effects of ones actions - at the margin - does not have a positive nor negative effect on the others.

- Here is a fully specified example.
- Consider a simple consumption labor supply problem, where agent j has preferences $U(c_j, e_j)$, and has the budget constraint

$$c_j = we_j + \frac{\Pi}{N}$$

  where $\frac{\Pi}{N}$ is the share of profits
- There are N agents and they are all the same
- We will focus on symmetric allocations $e_j = e \ \forall j$
- There is a competitive firm with technology $F(Ne)$
- Firm profit maximisation leads to

$$F'(Ne) = w \text{ or equivalently } w = w(Ne)$$

- The consumption good is the numéraire

- In this setup, the V function is

$$V(e_j, e) = U\left(w(Ne)e_j + \frac{\Pi(N, e)}{I}, e_j\right)$$

- The individual FOC is

$$wU_1 + U_2 = 0$$

- Let us now consider the social optimum
- We have $V(e,e) = U\left(w(Ne)e + \frac{\Pi(N,e)}{I}, e\right)$
- The Planner FOC is

$$wU_1 + U_2 + \sum_{j=1}^{N} U_1 \left[\frac{\partial w}{\partial e}e + \frac{(F'(Ne-w)\frac{\partial Ne}{\partial e}}{N} - \frac{\partial w}{\partial e}\frac{Ne}{N}\right] = 0$$

- Because $F'(Ne) = w$, this is

$$wU_1 + U_2 + \sum_{j=1}^{N} U_1 \left[\frac{\partial w}{\partial e}e - \frac{\partial w}{\partial e}\frac{Ne}{N}\right] = 0$$

which rewrites

$$wU_1 + U_2 + \sum_{j=1}^{N} U_1 \left[\frac{\partial w}{\partial e}(e-e)\right] = 0$$

- $$wU_1 + U_2 + \underbrace{\sum_{j=1}^{N} U_1 \left[\frac{\partial w}{\partial e}(e - e) \right]}_{0} = 0$$
- So that we are back to the competitive equilibrium condition.
- As we see, the actions of the others have an impact on one's payoff V, but nevertheless there are no inefficiencies at the Walrasian equilibrium.

- This setup extends naturally to situation with imperfect information.
- For example,the θ in $V(e_j, e; \theta)$ may be a latent variable, which is only observed with error by agents.
- This is the global games framework.
- Some of the insights from the global games setup
  - × Imperfect information may eliminate multiple equilibria.
  - × Imperfect information affects the strength of complementarities (now reaction is a function of the expectation of the others actions, which depends on one ?s expectations of others expectation). This can cause expectation to play the role of a state variable
  - × This may allow for shocks to higher order beliefs (an example is Angeletos & La'O (2013))

- N players
- Each agent accumulates E_i by playing e_i
- Decision rule and law of motion for E are

$$E_{it+1} = (1-\delta)E_{it} + e_{it} \tag{1}$$

$$e_{it} = \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + F(e_t) \tag{2}$$

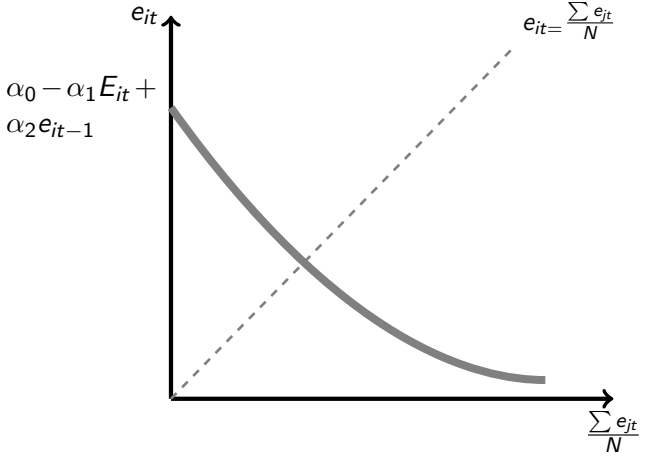
  with $e_t = \frac{\sum e_{jt}}{N}$
- all parameters are positive, $\delta < 1$, $\alpha_1 < 1$, $\alpha_2 < 1$.
- $\alpha_1 E_{it}$: Optimal size argument
- $\alpha_2 e_{it-1}$: Adjustment cost argument
- $F(e_t)$: Strategic Interactions
  - $F' > 0$: strategic complementarities
  - $F' < 0$: strategic substitutabilities

$$\begin{aligned} E_{it+1} &= (1-\delta)E_{it} + e_{it} \\ e_{it} &= \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + F(e_t) \end{aligned}$$

- Define (E^s, e^s) as the steady state of the linear model where $F(\cdot) = 0$
- Normalize $F(e^s) = 0$

2. A framework for Dynamics with Strategic Interactions

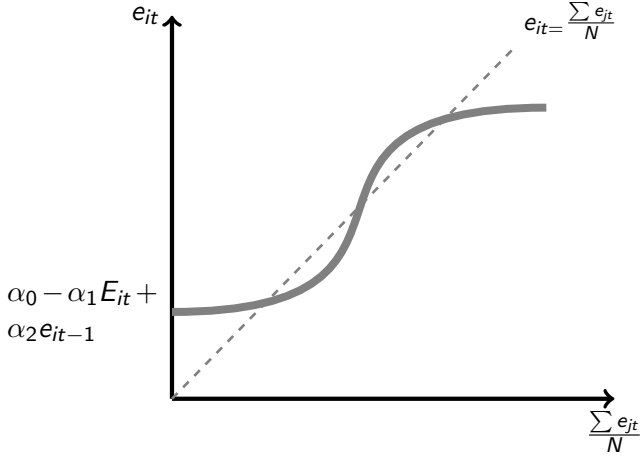
"Best response" rule for a given history - Strategic substitutability



$$e_{it} = \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + F\left(\frac{\sum e_{jt}}{N}\right)$$

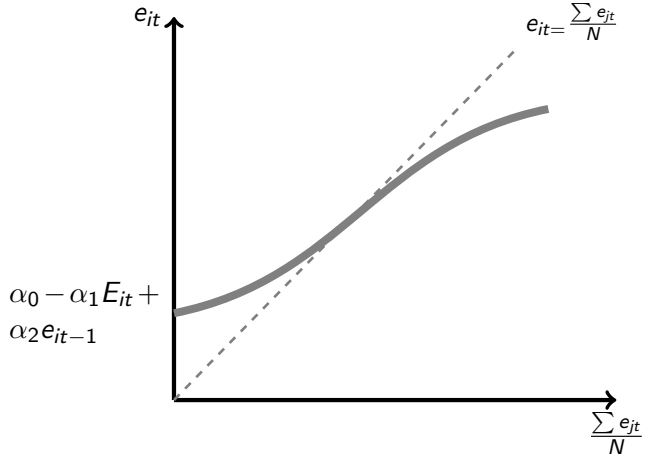
2. A framework for Dynamics with Strategic Interactions

"Best response" rule for a given history - Multiple Equilibria



$$e_{it} = \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + F\left(\frac{\sum e_{jt}}{N}\right)$$

- We restrict to the case where $F'(\cdot) < 1$, so that there are never multiple equlibria *within* period t



- Assume $\alpha_2 = 0$
- Then the model is

$$\begin{aligned} E_{it+1} &= (1-\delta)E_{it} + e_{it} \\ e_{it} &= \alpha_0 - \alpha_1 E_{it} + F(e_t) \end{aligned}$$

- It boils down to one order one dynamic equation in E_{it}

$$E_{it+1} = \alpha_0 + (1-\delta-\alpha_1)E_{it} + F(E_{t+1} - (1-\delta)E_t)$$

- So that, for symmetrical allocations

$$E_{t+1} = \alpha_0 + (1-\delta-\alpha_1)E_t + F(E_{t+1} - (1-\delta)E_t)$$

- Assume no strategic interactions : $F(\cdot) \equiv 0$

$$E_{t+1} = \alpha_0 + (1 - \delta - \alpha_1)E_t$$

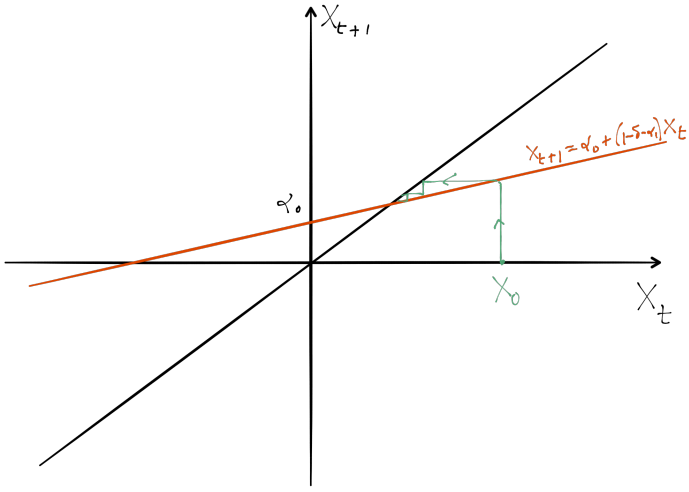
- Note that $(1 - \delta - \alpha_1)$ can be either > 0 or < 0.

Proposition 1

The Economy is globally stable as $|1 - \delta - \alpha_1| < 1$

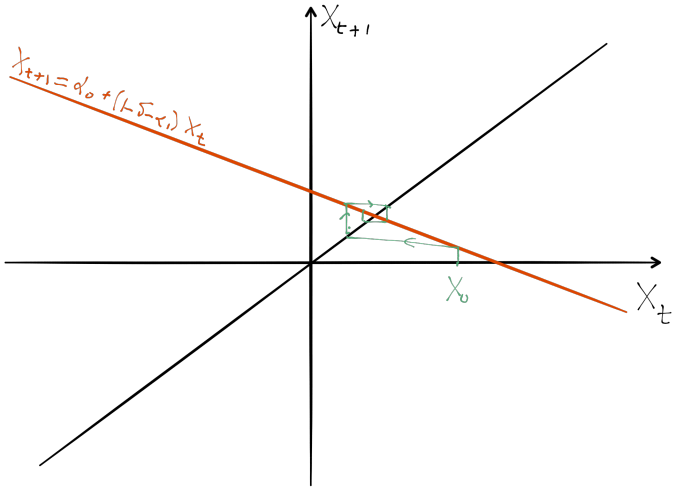
2. A framework for Dynamics with Strategic Interactions

One-Dimension Case



2. A framework for Dynamics with Strategic Interactions

One-Dimension Case



- Reintroduce strategic interactions

$$E_{t+1} = \alpha_0 + (1 - \delta - \alpha_1)E_t + F(E_{t+1} - (1 - \delta)E_t)$$

- Linearize around the steady state E^s

$$E_{t+1} = (\alpha_0 + \delta E^s) + (1 - \delta - \alpha_1)E_t + F'(E^s)(E_{t+1} - (1 - \delta)E_t)$$

- so that

$$E_{t+1} = \frac{\alpha_0 + \delta E^s}{1 - F'(E^s)} + \frac{((1 - \delta)(1 - F'(E^s)) - \alpha_1)}{1 - F'(E^s)} \quad E_t$$

$$E_{t+1} = \frac{\alpha_0 + \delta E^s}{1 - F'(E^s)} + \frac{((1-\delta)(1-F'(E^s)) - \alpha_1)}{1 - F'(E^s)} \quad E_t$$

Proposition 2

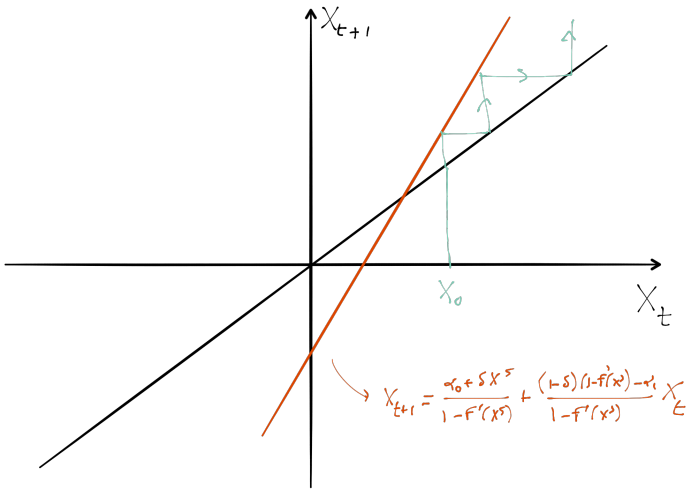
With strategic substitutability ($F'(\cdot) < 0$), the economy is always stable

Proposition 3

With strategic complementarity ($F'(\cdot) > 0$), there is always a level of complementarity smaller than 1 such that the SS is (locally) unstable.

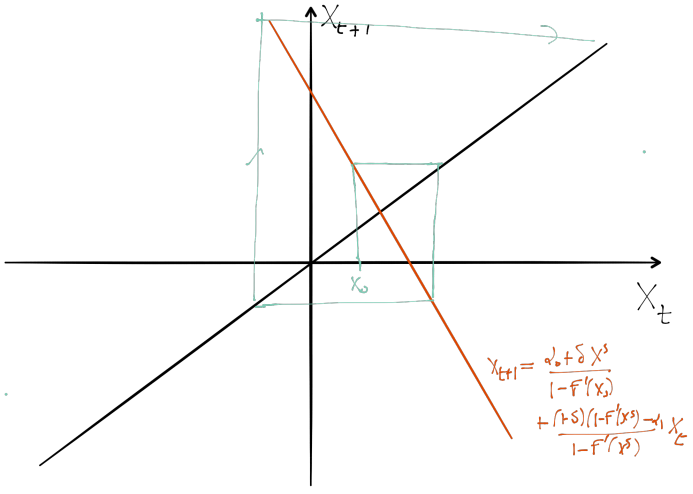
2. A framework for Dynamics with Strategic Interactions

One-Dimension Case

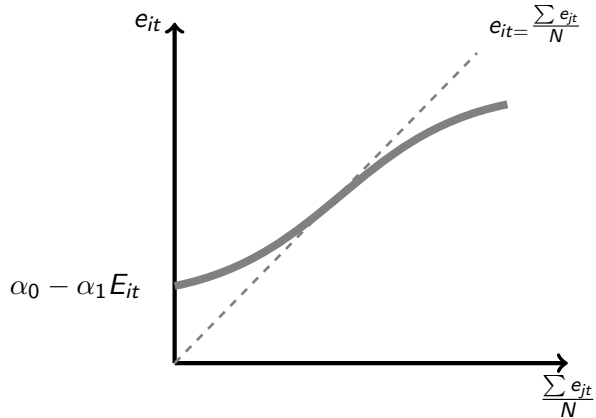


2. A framework for Dynamics with Strategic Interactions

One-Dimension Case



- The SS E^s becomes locally unstable
- "Real data" does not look explosive
- Let's assume that strategic complementarities dies out when the economy is far above of below E^s
- F is "S-shaped"



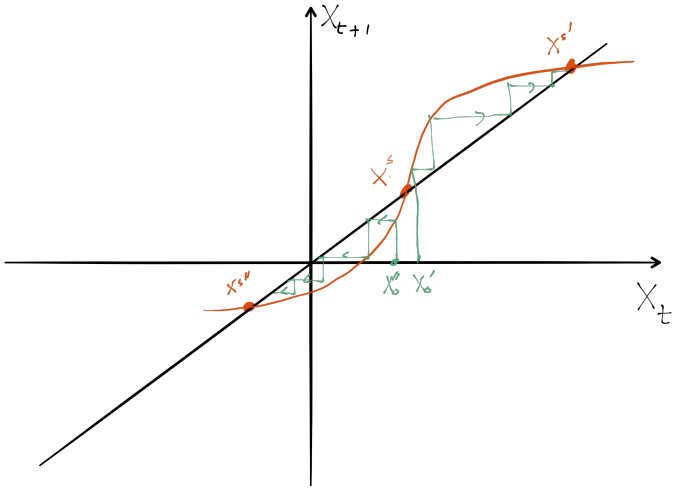
- Assume for simplicity that F is symmetric wrt E^s

$$E_{t+1} = \frac{\alpha_0 + \delta E^s}{1 - F'(E^s)} + \frac{((1-\delta)(1-F'(E^s)) - \alpha_1)}{1 - F'(E^s)} \quad E_t$$

- The dynamics of the economy will very much depend on whether $\frac{((1-\delta)(1-F'(E^s))-\alpha_1)}{1-F'(E^s)}$ is positive or negative
  - × Positive : Hysteresis
  - × Negative : Limit cycle
- Need to think of environments that makes this slope is positive or negative

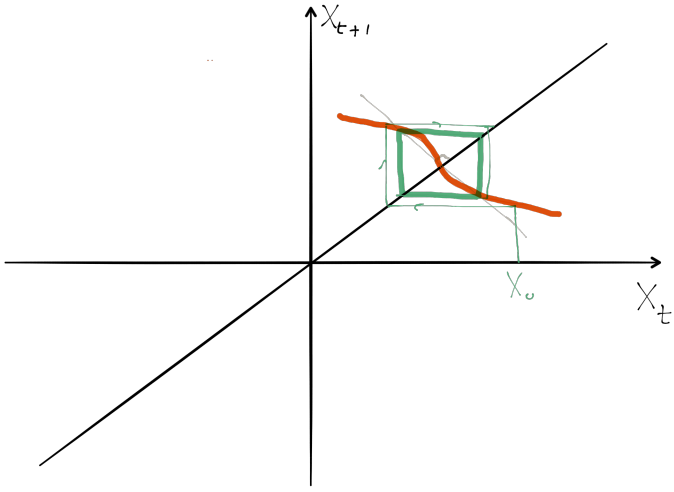
2. A framework for Dynamics with Strategic Interactions

One-Dimension Case



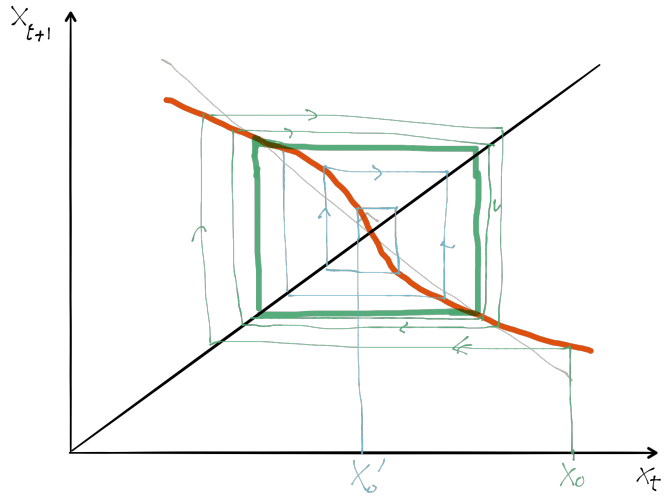
2. A framework for Dynamics with Strategic Interactions

One-Dimension Case



2. A framework for Dynamics with Strategic Interactions

One-Dimension Case



- Problem with one dimension model : the dynamics does not look like a business cycle
- E_t is *not* positively correlated.
- This needs not to be true in the more general model where $\alpha_2 > 0$

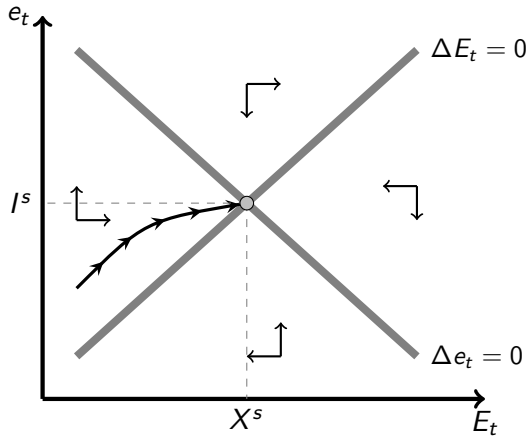
$$\begin{aligned} E_{it+1} &= (1-\delta)E_{it} + e_{it} \\ e_{it} &= \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + F(e_t) \end{aligned}$$

- Without strategic interactions ($F(\cdot) \equiv 0$)
- When all agent behave symmetrically :

$$\begin{pmatrix} e_t \\ E_t \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha_2 - \alpha_1 & -\alpha_1(1-\delta) \\ 1 & 1-\delta \end{pmatrix}}_{M_L} \begin{pmatrix} e_{t-1} \\ E_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha_0 \\ 0 \end{pmatrix}$$

Proposition 4

Both eigenvalues of the matrix M_L lie within the unit circle. Therefore, the system is stable.



Here we have assumed that the two eigenvalues are real and positive.

- Math
- Dynamics is

$$\begin{pmatrix} e_t \\ E_t \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha_2 - \alpha_1 & -\alpha_1(1-\delta) \\ 1 & 1-\delta \end{pmatrix}}_{M_L} \begin{pmatrix} e_{t-1} \\ E_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha_0 + F(e_t) \\ 0 \end{pmatrix}$$

- Local dynamics is

$$\begin{pmatrix} e_t \\ E_t \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{\alpha_2-\alpha_1}{1-F'(I^s)} & -\frac{\alpha_1(1-\delta)}{1-F'(I^s)} \\ 1 & 1-\delta \end{pmatrix}}_{M} \begin{pmatrix} e_{t-1} \\ E_{t-1} \end{pmatrix} + \begin{pmatrix} \left(1 - \frac{\alpha_2-\alpha_1}{1-F'(I^s)}\right) I^s + \left(\frac{\alpha_1(1-\delta)}{1-F'(I^s)}\right) X^s \\ 0 \end{pmatrix}$$

- When $F'(I^s)$ varies from $-\infty$ to 1, eigenvalues of M vary

Proposition 5

As $F'(I^S)$ varies from 0 to $-\infty$, the eigenvalues of M always stay within the unit circle and therefore the system remains locally stable.

Proposition 6

As $F'(I^s)$ *varies from* 0 *towards* 1, *the dynamic system will become locally unstable.*

(bifurcation)

- Here we can have a limit cycle with persistence
- Consider the steady state (E^s, I^s)
- Strategic complementarities : centrifugal force that pushes away from the steady state when close to.
- Accumulated variable X : centripetal force that pushes towards the steady state when away from.
- The steady state locally unstable, but forces push the economy back to the steady state when it is further from it.
- It is quite intuitive why a limit cycle occurs when the steady state moves from stable to unstable

FIGURE 1 – Stable Steady State

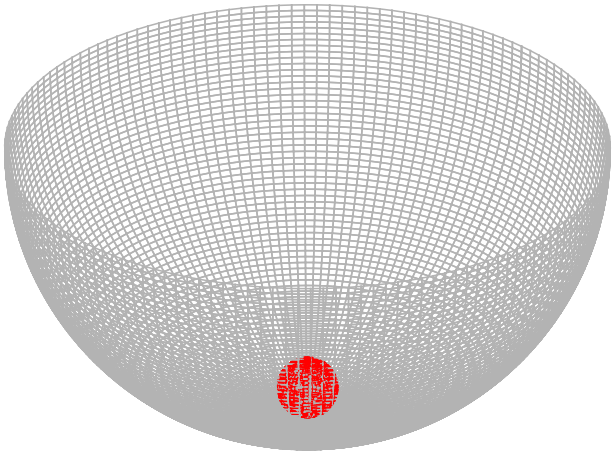


FIGURE 2 – HOPF Supercritical bifurcation : Attractive Limit Cycle

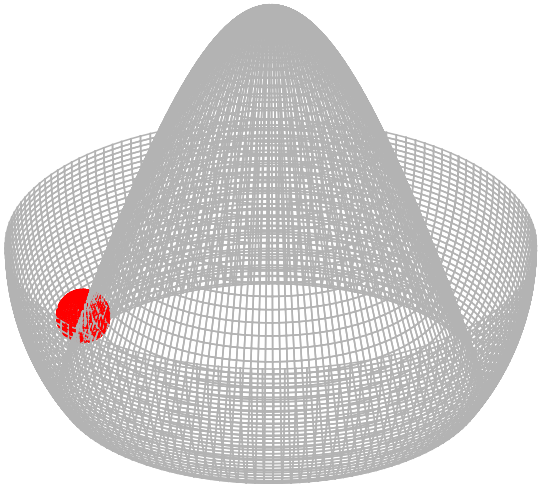


FIGURE 3 – Unstable Steady State

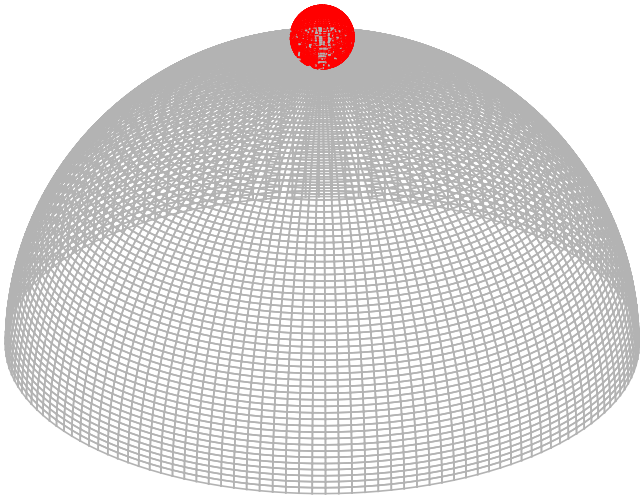
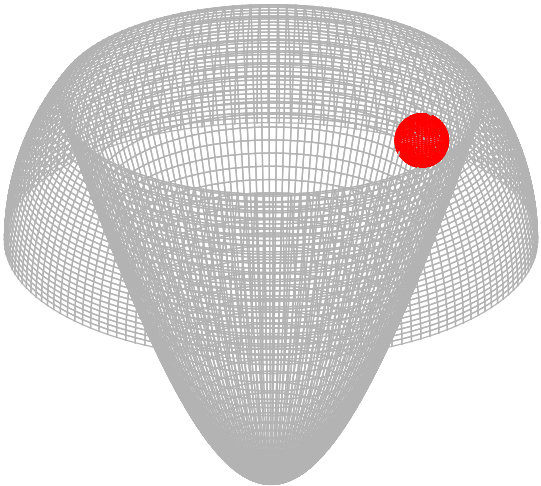
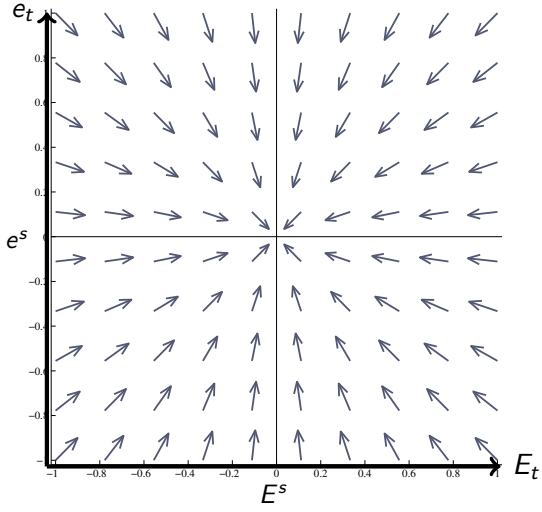


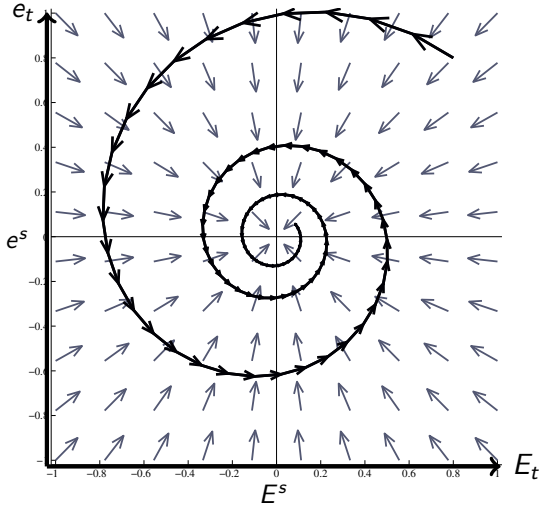
FIGURE 4 – HOPF Subcritical bifurcation : Repulsive Limit Cycle



2. A framework for Dynamics with Strategic Interactions

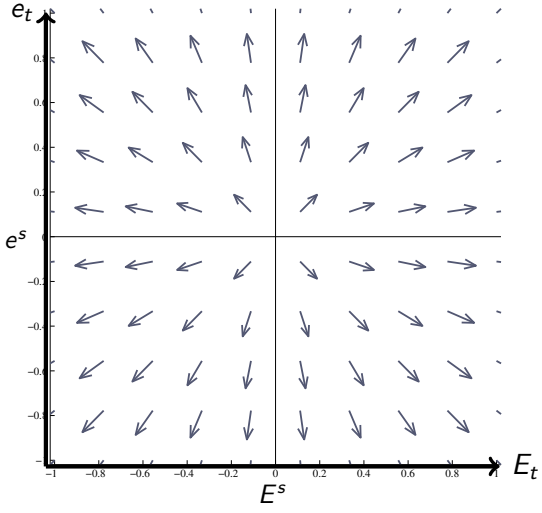
Two-Dimension Case - Global stability




e_t
e^s
E_t
E^s
−1
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1

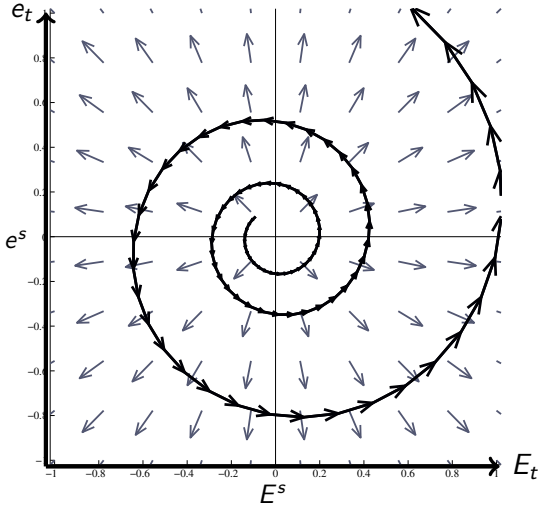
2. A framework for Dynamics with Strategic Interactions

Two-Dimension Case - Global instability



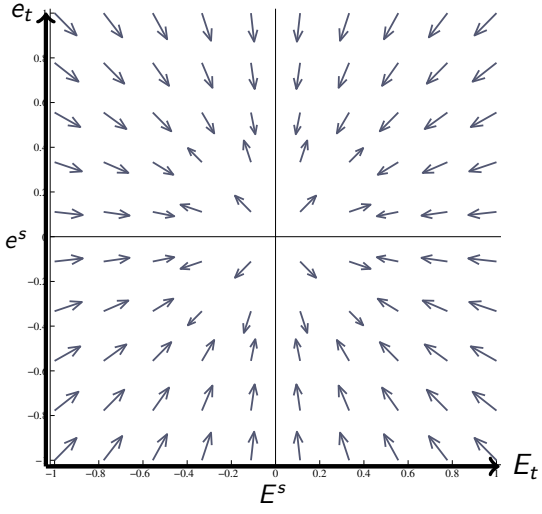
2. A framework for Dynamics with Strategic Interactions

Two-Dimension Case - Global instability



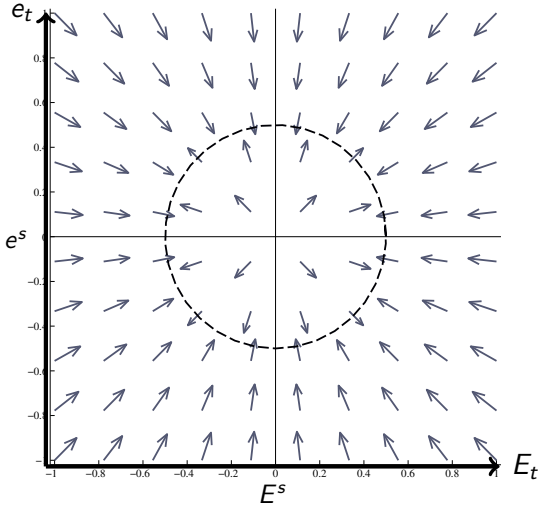
2. A framework for Dynamics with Strategic Interactions

Two-Dimension Case - Stable limit cycle

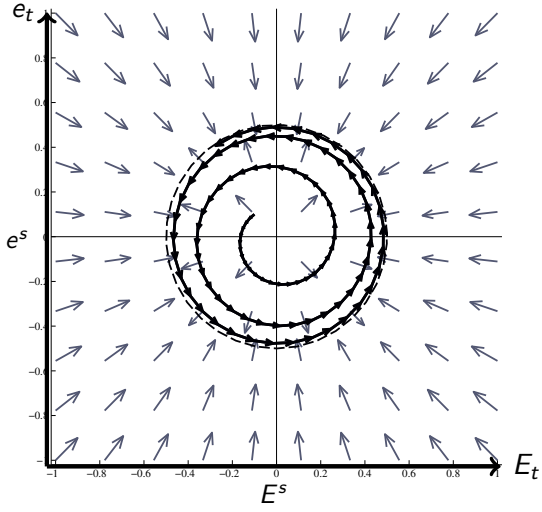


2. A framework for Dynamics with Strategic Interactions

Two-Dimension Case - Stable limit cycle

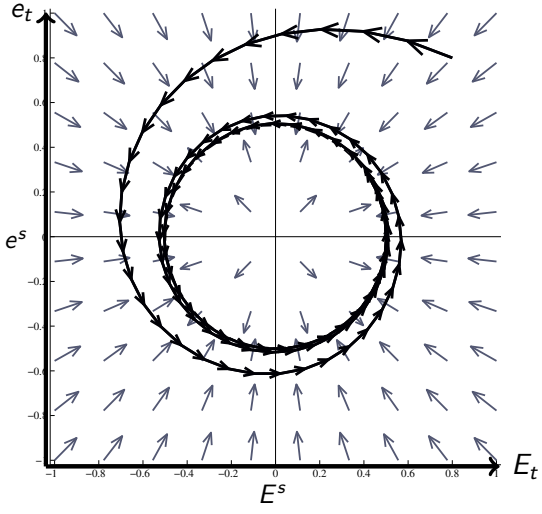


Two-Dimension Case - Stable limit cycle



2. A framework for Dynamics with Strategic Interactions

Two-Dimension Case - Stable limit cycle



- Discrete time version of the HOPF theorem.
- Consider the 2-dimensional smooth system

$$Y_{t+1} = F(Y_t; \alpha)$$

- Assume : Steady State for $\alpha = 0$: $F(\vec{0}; 0) = 0$
- Assume : Jacobian of F at (0,0) has two eigenvalues $(1 + \alpha)e^{\pm i\theta_0}$
- That system has a bifurcation at $\alpha = 0$
- *i.e.* the Steady State changes stability with α
- Then, the system $Y_{t+1} = F(Y_t; \alpha)$ has a limit cycle near the origin.
- Note : other possibilities is that one eigenvalue is 1 or -1.

- Nice theorem : we simply have to look at the *linearized* dynamics to prove existence of a limit cycle
- The parameter α is here the degree of strategic complementarities at the steady state $F'(I^s)$
- It is quite intuitive why a limit cycle occurs when the steady state moves from stable to unstable

- 3 types of bifurcation
  - × Fold bifurcation : appearance of an eigenvalue equal to 1,
  - × Flip bifurcation : appearance of an eigenvalue equal to -1
  - × HOPF bifurcation : appearance of two complex conjugate eigenvalues of modulus 1.
- We are interested in HOPF bifurcation because the limit cycle will be "persistent"

Proposition 7

As $F'(I^s)$ varies from 0 towards 1, the dynamic system will become unstable and :

- *If $\alpha_2 > \alpha_1/(2-\delta)^2$, then a* Hopf *(*Neimark-Sacker*) bifurcation will occur.*
- *If $\alpha_2 < \alpha_1/(2-\delta)^2$, then a flip bifurcation will occur.*

- In the case of flip and HOPF bifurcation, a limit cycle appears
- In case of HOPF, the cycle is "smooth"
- Condition for an HOPF bifurcation :

$$e_{it} = \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + F\left(\frac{\sum e_{jt}}{N}\right)$$

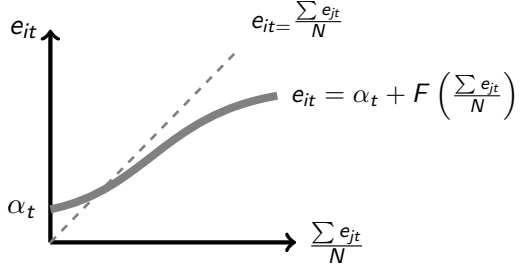
  - × as δ approaches 0, $\alpha_2 > 1/4$
  - × in general, what is needed is δ not too large and α_2 large enough

- In the case of the HOPF bifurcation, the limit cycle can be attractive (*the bifurcation is supercritical*) or repulsive (*the bifurcation is subcritical*)

Proposition 8

If $F'''(I^s)$ is sufficiency negative, then the HOPF *bifurcation will be supercritical. Therefore, the limit cycle is attractive.*

-
- $F''' < 0$ corresponds to an $S-$*shaped* reaction function



-

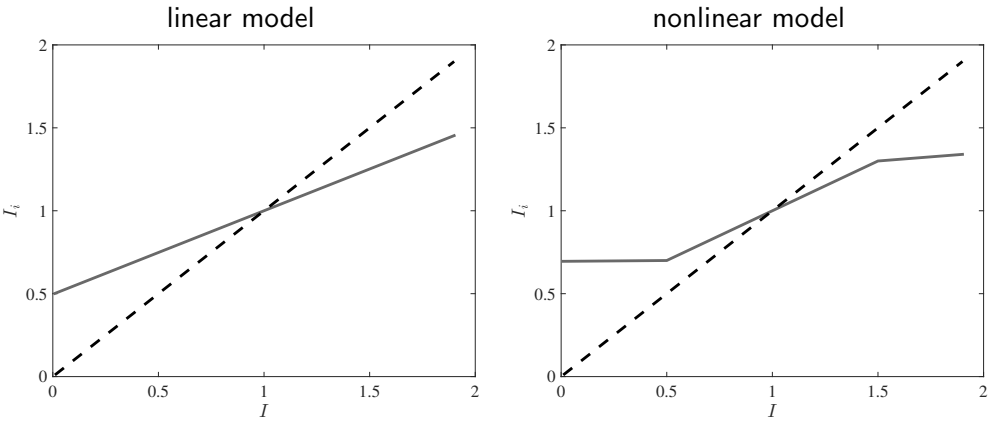
$$\alpha_t = \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1}$$

3. Quantitative Reduced Form Limit Cycle Model

Model

- $E_{it+1} = (1-\delta)E_{it} + e_{it}$
- $e_{it} = \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + F\left(\frac{\sum e_{it}^s}{N}, u_t\right)$
- $F(e_t, u_t) = \widetilde{F}(e_t) + u_t.$
- $u_t = \rho u_{t-1} + \varepsilon_t$
- $\widetilde{F}$: S-shaped and piecewise linear

FIGURE 5 – Best response rules in the numerical example



- Steady state is $e_{SS} = 1$, $E_{SS} = 10$
- Deterministic simulation : let $E_0 = 8$, $e_0 = 1$

FIGURE 6 – Deterministic simulation

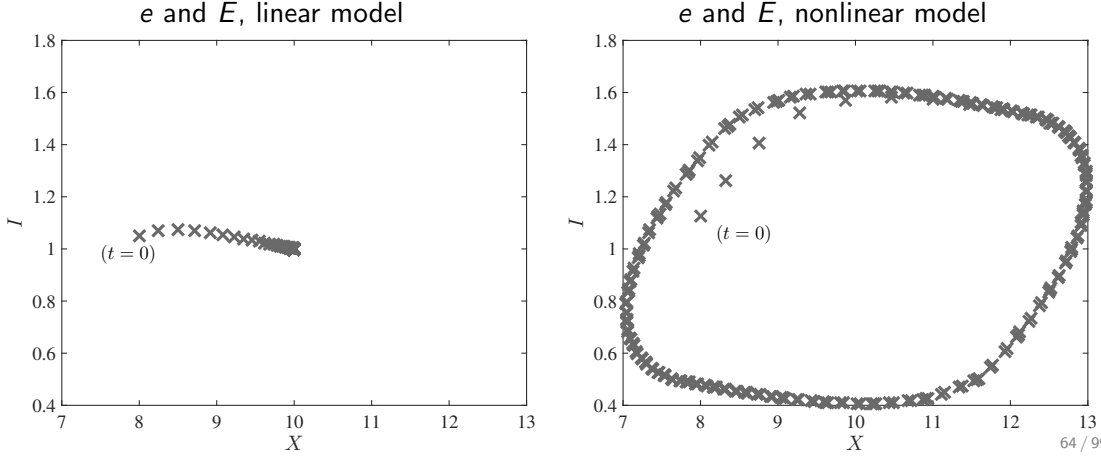


FIGURE 7 – Deterministic simulation

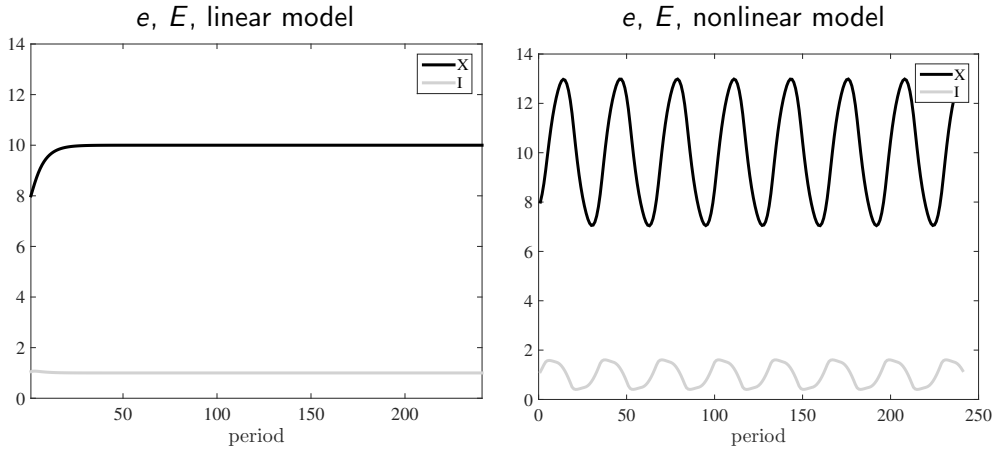


FIGURE 8 – Deterministic simulation of the nonlinear model

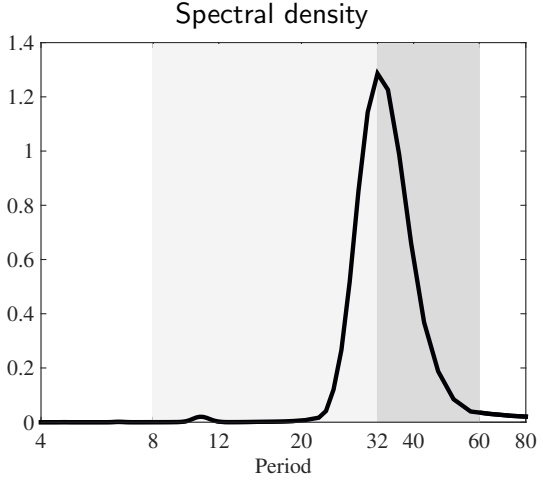


FIGURE 9 – One stochastic simulation

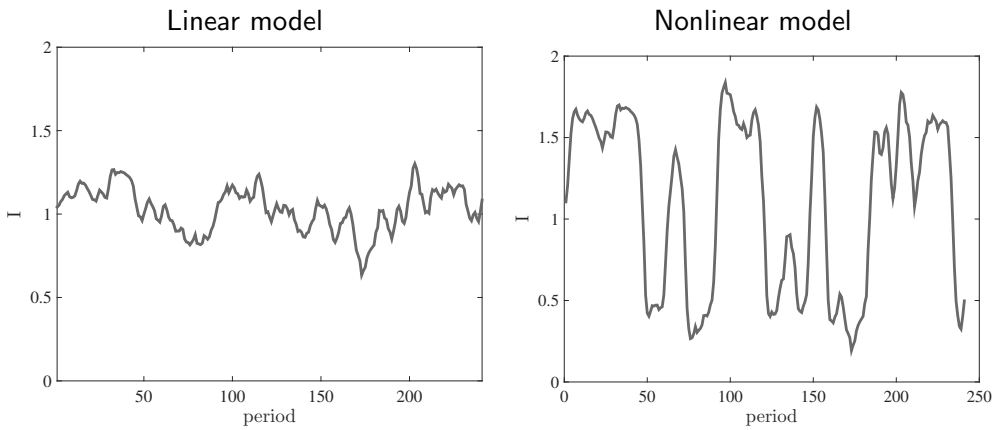
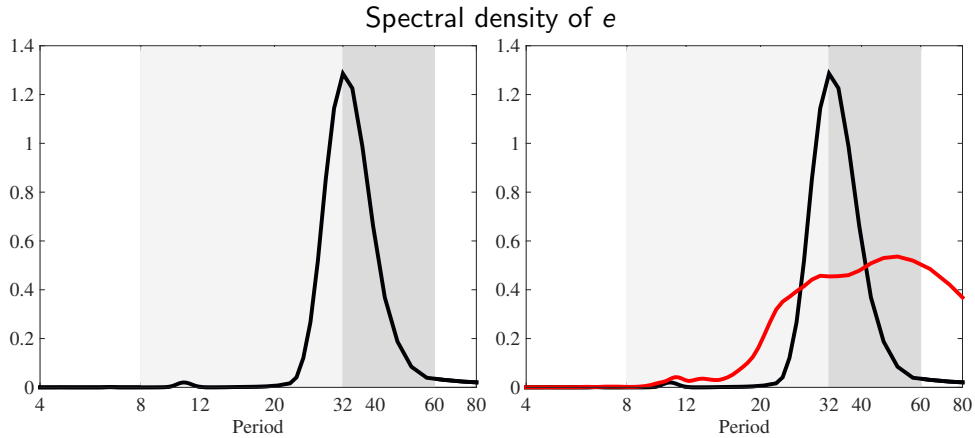


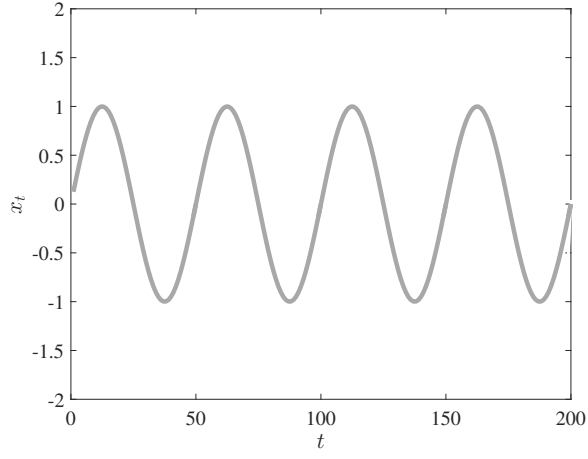
FIGURE 10 – Deterministic and Stochastic simulation of the nonlinear model



3. Quantitative Reduced Form Limit Cycle Model

What the results are not

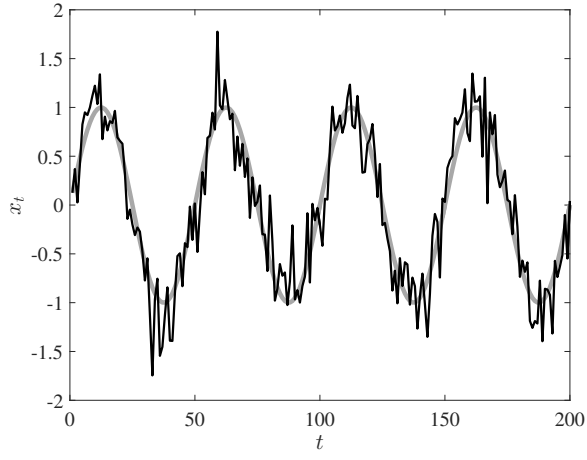
FIGURE 11 – $x_t = \sin(\omega t)$



3. Quantitative Reduced Form Limit Cycle Model

What the results are not

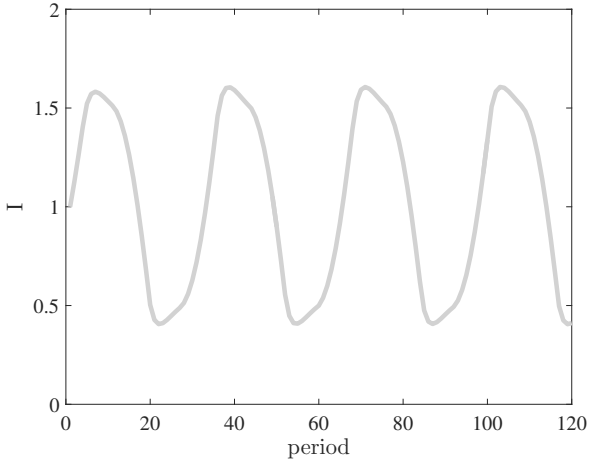
FIGURE 12 – $x_t = \sin(\omega t) + u_t$



3. Quantitative Reduced Form Limit Cycle Model

What the results are

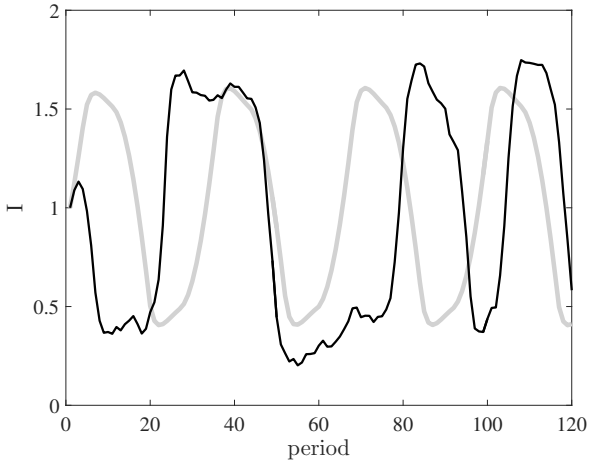
FIGURE 13 – Reduced Form Model



3. Quantitative Reduced Form Limit Cycle Model

What the results are

FIGURE 14 – Reduced Form Model



4. Adding forward looking elements

- $$e_{it} = \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + \alpha_3 E_t[e_{it+1}] + F\left(\frac{\sum e_{jt}}{N}\right)$$
- with accumulation remaining the same

$$E_{it} = (1-\delta)E_{it} + e_{it}$$

- Restrict attention to situations where this system is saddle path stable absent of complementarities.
- The local dynamics is described by the 3 eigenvalues of the linearized system

Proposition 9

If unique steady state, then no indeterminacy nor Fold bifurcations (always one positive and greater than one eigenvalue)

Proposition 10

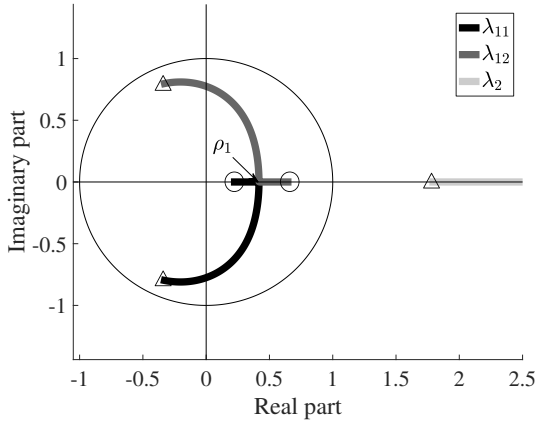
If α_2 (sluggishness) sufficiently large, then the two other eigenvalues will become complex and

- *either will stay inside the unit disk (non locally explosive cycles)*
- *or will exit the unit disk (HOPF bifurcation and limit cycles)*

4. Adding forward looking elements

Set of results when one increases ρ

FIGURE 15 – Increasing complementarities ρ : local stability



4. Adding forward looking elements

Set of results when one increases ρ

FIGURE 16 – Increasing complementarities ρ : local instability and limit cycle

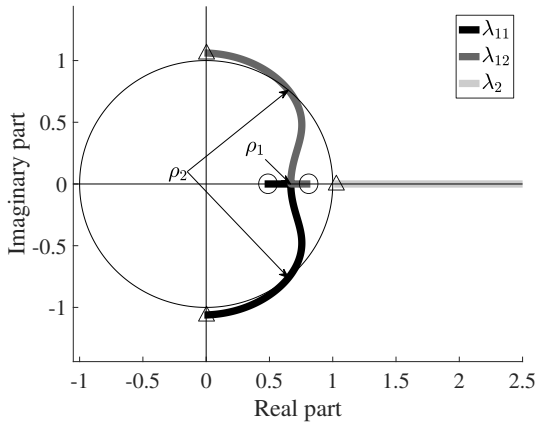


FIGURE 17 – A Saddle Limit Cycle

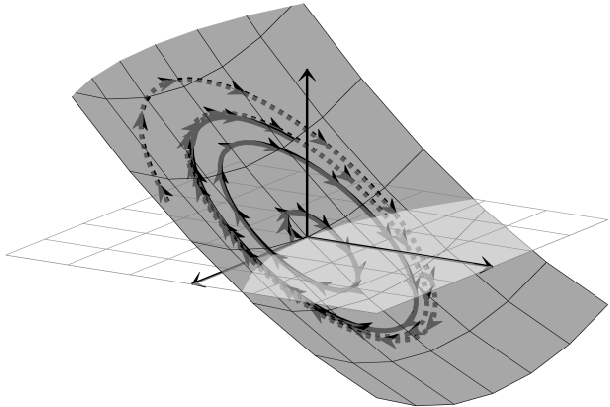


FIGURE 18 – Explosive path

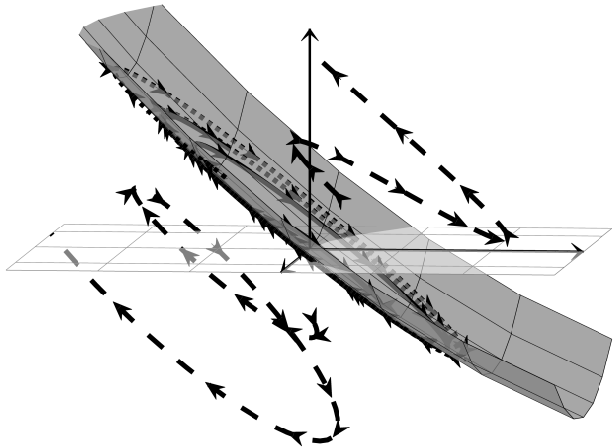
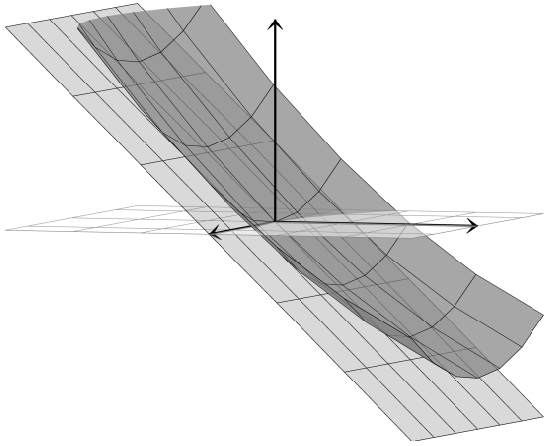


FIGURE 19 – Linear and non linear stable manifolds



4. Adding forward looking elements

- Weak complementarities + dynamics create cycles
- It can also create local instability and limit cycles
- One should be aware that estimating linear models is likely to biaise against this mechanism as one rules out local instability.

5. A NK Model with Financial Frictions

$$\begin{cases} e_{it} &= \alpha_0 - \alpha_1 E_{it} + \alpha_2 e_{it-1} + \alpha_3 E_t[a_{it+1}] + F\left(\frac{\sum e_{jt}}{N}\right) \\ E_{it} &= (1-\delta)E_{it} + e_{it} \end{cases}$$

- Stylized NK model which is extended to allow for the forces highlighted in our general structure.
- We add accumulation of durable-housing goods and habit persistence : *accumulation and sluggishness*
- Financial frictions : households face unemployment risk and cannot fully insure against it $\rightsquigarrow$ counter-cyclical risk premium $\rightsquigarrow$ *complementarities*
- Estimate parameters based on spectrum observations and higher moments. (use perturbation method and indirect inference)
- AR(1) discount factor shock, with persistence ρ

5. A NK Model with Financial Frictions

Allowing for durable goods and accumulation

- The representation household who can buy/rent consumption services from the market.
- Households consumption services are supplied by firms and come from either the production of new goods or from the service flow of existing durables.

$$C_t = sX_t + F(L_t\theta_t)$$

where X_t is the stock of durable goods and $F(l_t\theta_t)$ is the current production of goods. The accumulation of durables is

$$X_{t+1} = (1-\delta)X_t + \psi F(L_t\theta_t)$$

where a fraction $\psi \leq (1-\delta)$ of newly produced goods become durables after one period.

- Households own the stock of durable-housing, rent it to firms, who supply back the consumption services as a composite good.

5. A NK Model with Financial Frictions

Risk premium & Financial Frictions

- Labor indivisibility $\rightsquigarrow$ some workers are unemployed
- Spending decisions are made before labor market outcome is known
- The large family pays only for a fraction of its unemployed members.
- The interest rate which household face is then equal to the policy rate plus a risk premium

$$(1 + r_t) = (1 + i_t + r_t^p)$$

- The risk premium is a decreasing function of the economic activity, where the output gap and employment gap are interchangeable

$$r_t^p = G(\ell_t)$$

- The interest premium is generically a non-linear function of activity.
- The households Euler will have the familiar form (assuming external habit)

$$U'(C_t - \gamma C_{t-1}) = \beta_t E_t U(C_{t+1} - \gamma C_t)(1 + r_t)$$

- Firms face stochastic opportunities to change price (Calvo)
- Have Taylor rule of form

$$i_t = E_t \pi_{t+1} + \phi E_t \ell_{t+1}$$

where ℓ_t is (log) employment or output gap, and $U(\cdot) = \ln(\cdot)$

- Estimate parameters of model by SMM
- Targets
  - × spectrum of hours worked on the frequencies 2-50
  - × spectrum of interest rate spread on the frequencies 2-50
  - × Set of other higher moments (correlation, kurtosis and skewness of hours and spread)
- We estimate the model
  - × Without complementarities (No Risk Pemium)
  - × With complementarities, but linear approx.
  - × With complementarities, but order 3 approx.

FIGURE 20 – Spectral Density, Level and Bandpass (2,100) to (2,200), Full Sample - Non Farm Business Hours

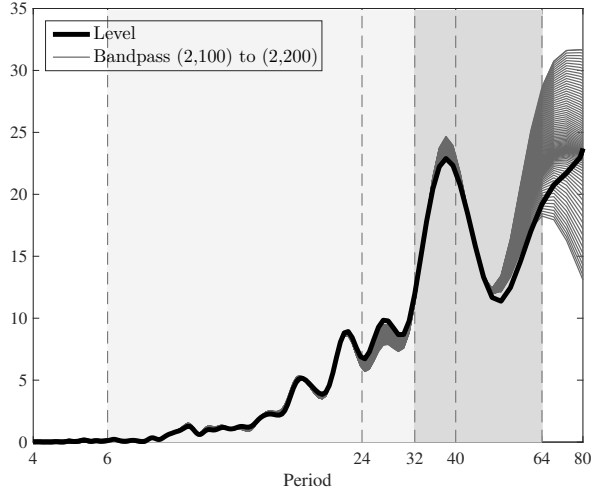


FIGURE 21 – Spectrum of Hours, No Risk Premium

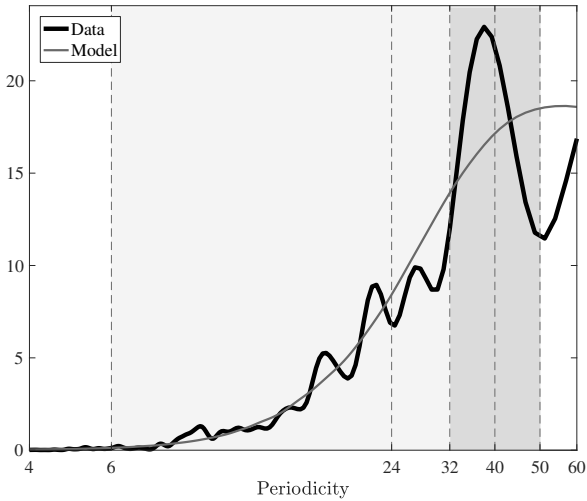


FIGURE 22 – Spectrum of Hours, Linear Risk Premium

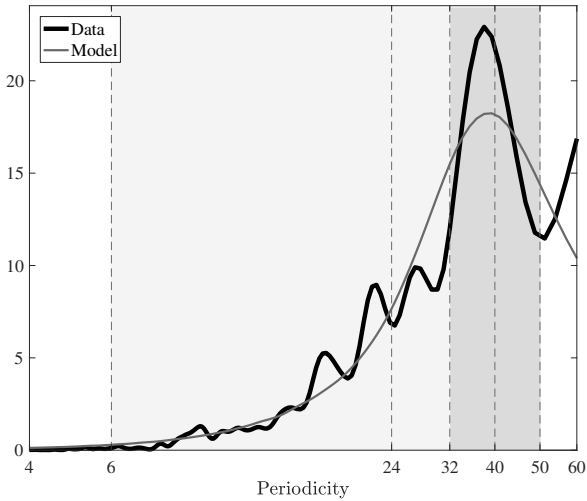


FIGURE 23 – Spectrum of Hours, Non-Lin. Risk Premium

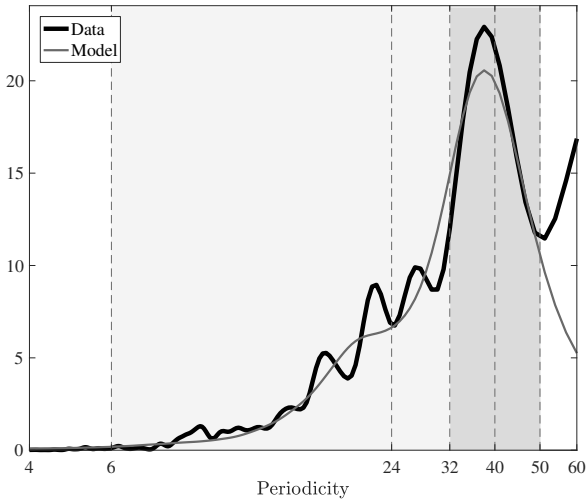


FIGURE 24 – Hours Spectrum in Smets & Wouters' Model

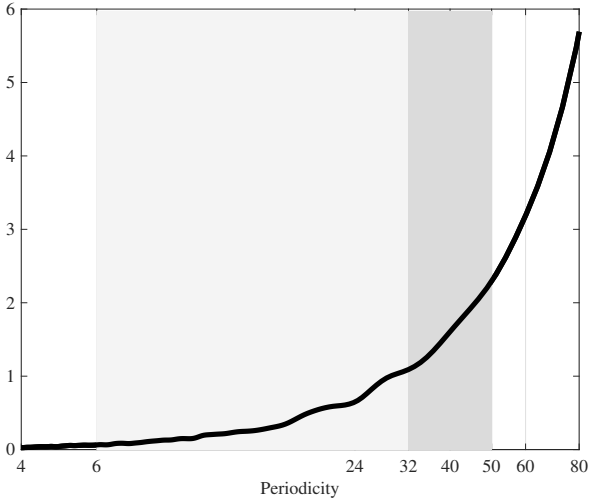


TABLE 1 – Parameter Estimates

		Non-Lin. RP	Lin. RP	No RP
ω	CRRA	0.2997	0.2408	0.2408*
γ	Habit	0.5335	0.5876	0.8346
ψ	Initial deprec.	0.4000	0.2994	0.1154
ϕ	Taylor rule	0.0421	0.0467	0.0000
R_2	Risk prem.	0.00017	—	—
R_3	Risk prem.	0.00006	—	—
$\bar{f}$	Debt backing	0.8668	0.8827	1*
b	Default cost	0.2870	0.3568	—
ρ	Autocorr.	-0.0000	0.1387	0.8346
σ	Innov. s.d.	0.0157	0.0272	0.0335
$s.d.(\mu)$	Uncond. s.d.	0.0157	0.0275	0.0609

TABLE 2 – Eigenvalues

	Non-Lin. RP	Lin. RP	No RP
λ_1, λ_2	$1.1032 \pm 0.2164i$	$0.9258 \pm 0.1372i$	0.8346, 0.8346
$\lvert\lambda_j\rvert$	1.1242	0.9359	0.8346

FIGURE 25 – Sample Draw for Hours

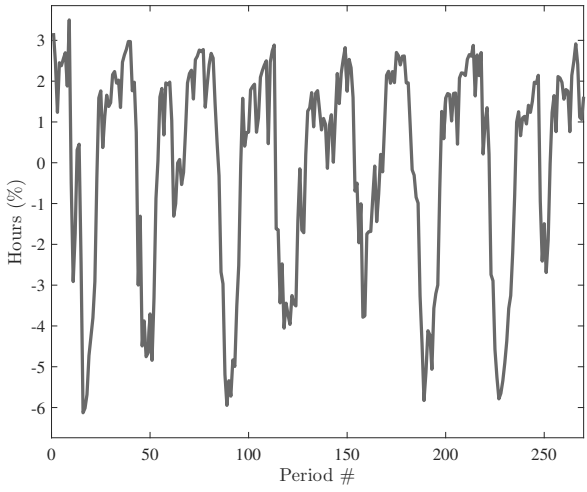


FIGURE 26 – Sample Draw for Hours, no shocks

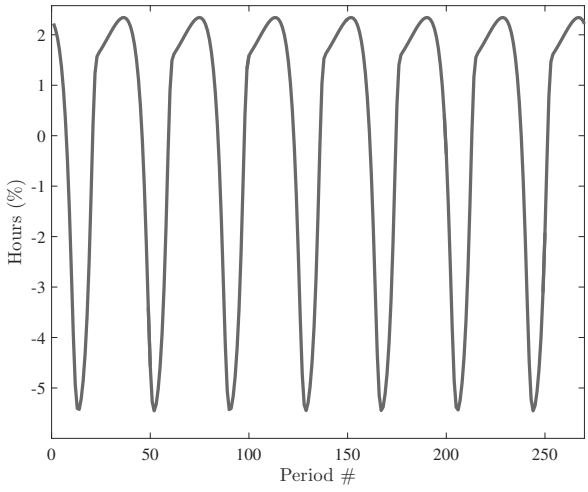
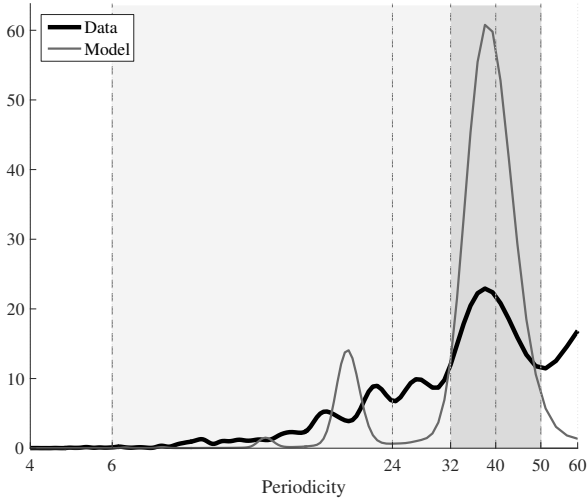


FIGURE 27 – Spectrum for Hours, no shocks



6. Results From Estimation

- Shocks are important for explaining the data.
- However, we estimate them to be i.i.d. in case with limit cycle.
  - × Almost all dynamics are internal.
  - × Main purpose : makes cycle *irregular*.
- Important : Adding shocks to a model with a limit cycle does not simply result in a predictable cycle with noise around it.
  - × Shocks have temporary amplitude effect, but "random-walk" effect on phase.
  - × i.e., permanently shift the whole cycle forward or backward in time.
  - × Phase of cycle completely unpredictable in the infinite future.

- Estimate the linear and non linear version over alternative frequencies : 2-32, 2-50, 2-60, 2-100.
- Main results
  - × over 2-32 : come back to case where the exogenous driving forces is the cycle
  - × Our results not changed much if go to 2-60 or 6-50, 6-60.. Almost all dynamics are internal
  - × However, if extend further, example 2-100, back to exogenous driving force dominating

6. Results From Estimation

Estimating over different frequency windows

FIGURE 28 – Estimation over 2-100

