**SUMMER TERM 2020**

# ECON0039: ADVANCED MACROECONOMICS
Solution

**Coursework assessment**

*All questions should be answered.*

*Part A carries 20% of the total mark, part B 40% and part C 40%. Questions B.1 and B.2 have the same weight.*

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## PART A : Inequalities

Consider an economy with aggregate capital stock $K_t$ and aggregate labour $L_t$. There is a representative (competitive) firm which produces output according to the production function $F(K_t, L_t)$. $F$ is a neoclassical production function with constant returns to scale. $Y_t$ denotes production net of depreciation. If $\widetilde{Y}_t = F(K_t, L_t)$ is gross production, then $Y_t = \widetilde{Y}_t - \delta K_t$, where $\delta$ is the depreciation rate. Denote by $r_t$ the cost of capital services and by $w_t$ the cost of labour services.

There are $n$ individuals indexed by $i$ in this economy. Each holds a capital stock $k_{it}$ and supplies $\ell_{it}$ unit of labour. Individual net income is given by $y_{it} = w_t\ell_{it} + r_t k_{it}$. Aggregate quantities are $Y_t = \sum_{i=1}^n y_{it}$, $K_t = \sum_{i=1}^n k_{it}$ and $L_t = \sum_{i=1}^n \ell_{it}$. The wealth of agent $i$ in period $t$ is $k_{it}$. Individual wealth accumulates according to the law of motion $\Delta k_{it} = k_{it+1} - k_{it} = y_{it} - c_{it} = w_t\ell_{it} + r_t k_{it} - c_{it}$.

We assume that the consumption function of an individual is

$$c_{it} = \overline{c} + \widehat{c}\, y_{it} + \widetilde{c}\, k_{it}$$

A.1 Using the profit maximisation first order conditions of the representative firm, show that, at the competitive equilibrium, $r_t$ and $w_t$ are functions of aggregate quantities $K_t$ and $L_t$.

The profit of the representative firm is $\widetilde{Y}_t - w_t L_t - r_t K_t$. As it is competitive, profit maximisation implies

$$\begin{aligned} w_t &= \frac{\partial F(K_t, L_t)}{dL_t}, \\ r_t &= \frac{\partial F(K_t, L_t)}{dK_t}. \end{aligned}$$

We observe that $w$ and $r$ depend only on aggregate quantities $K$ and $L$. Because labor and capital are homogenous inputs, it does not matter who owns capital and who supplies labour.

A.2 Discuss the consumption function, and the parameters $\overline{c}$, $\widehat{c}$ and $\widetilde{c}$.

Consumption depends linearly on current income $y$ and also on the current stock of the accumulated wealth $k$. $\overline{c}$ and $\widehat{c}$ denote the marginal (and average becasue of linearity) propensities to consume out of income and accumulated wealth, respectively. If $\widetilde{c} \geq 0$ , this parameter may be naturally interpreted as subsistence consumption. The inclusion of the wealth component means that, *for the same income*, people with higher wealth consume more. This might have to do with wealthy not needing to save a large share of their income for precautionary motives, because they already have a large buffer of asset. Finally, note that consumption is linear in income and wealth. That will prove to be a crucial assumption for aggregation and inequalities.

A.3 Show that this formulation encompasses the savings behaviour assumed in the Solow growth model as a special case. In the Solow model, savings equal a constant fraction $s$ of gross income. For which values of $\overline{c}$, $\widehat{c}$ and $\widetilde{c}$ do we obtain the Solow saving behaviour?

In the Solow growth model, savings equal a constant fraction $s$ of gross income. Since gross income $\widetilde{y}$ is given by $\widetilde{y} = y + \delta k$ where $\delta$ is the rate of depreciation, consumption of an individual is $c_{it} = (1-s)y_{it} + (1-s)\delta k_{it}$. Hence savings behaviour in the Solow model is the special case of that model analysed in the text when $\overline{c} = 0$, $\widehat{c} = 1 - s \in [0,1]$ and $\widetilde{c} = (1-s)\delta$.

A.4 Write the law of motion of $k_{it}$ and use the consumption function to derive an equation that relates $\frac{\Delta k_{it}}{k_{it}}$ to $r_t$, $w_t \ell_{it}$ and $k_{it}$.

The law of motion of $k_{it}$ is

$$\Delta k_{it} = w_t \ell_{it} + r_t k_{it} - c_{it}.$$

Using the consumption function, we obtain

$$\Delta k_{it} = \left((1-\widehat{c})\, r_t - \widetilde{c}\right) k_{it} + (1-\widehat{c})\, w_t \ell_{it} - \overline{c}.$$

Dividing both sides by $k_{it}$, we obtain

$$\frac{\Delta k_{it}}{k_{it}} = (1-\widehat{c})\, r_t - \widetilde{c} + \frac{(1-\widehat{c})\, w_t \ell_{it} - \overline{c}}{k_{it}}.$$

As one can see, the growth rate of a household wealth is a linear function of the average propensity to save capital income and on the wealth component of the consumption function. Savings out of labor income contribute less to wealth growth when wealth is high.

A.5 Write the law of motion of aggregate capital by aggregating individual capital law of motions. Do aggregate quantities depend on the distribution of $k$ and $\ell$ in the population? Why?

We have already derived

$$\Delta k_{it} = \big((1-\widehat{c})\, r_t - \widetilde{c}\big) k_{it} + (1-\widehat{c})\, w_t \ell_{it} - \overline{c}. \qquad (\star)$$

Summing over all individuals:

$$\sum_{i=1}^{n} \Delta k_{it} = \big((1-\widehat{c})\, r_t - \widetilde{c}\big) \sum_{i=1}^{n} k_{it} + (1-\widehat{c})\, w_t \sum_{i=1}^{n} \ell_{it} - n\overline{c},$$

which gives

$$\Delta K_t = \big((1-\widehat{c})\, r_t - \widetilde{c}\big) K_t + (1-\widehat{c})\, w_t L_t - n\overline{c}.$$

Recall that $w$ and $r$ depend only on aggregate quantities. The accumulation of aggregate capital depends only on aggregate capital and aggregate labor supply. There are two reasons why we obtain that result. First, all agents face the same return on factors ($w$ and $r$). Second, the consumption function is linear, so that we have been able to factorise out propensities to consume.

A.6 Assume that all individuals work an equal and constant amount of time, so that $\ell_{it} = \ell$. What is the condition under which wealth inequalities will be reduced with time? Note that wealth inequalities decrease with time if a wealth-poor individual accumulates wealth faster than a wealth-rich individual. Discuss that condition.

We have in that case

$$\frac{\Delta k_{it}}{k_{it}} = (1-\widehat{c})\, r_t - \widetilde{c} + \frac{(1-\widehat{c})\, w_t \ell - \overline{c}}{k_{it}}.$$

Higher wealth levels grow slower if $(1-\widehat{c})\, w_t \ell - \overline{c} > 0$. Hence, there is convergence toward more equality if the economy satisfies the condition

$$(1-\widehat{c})\, w_t \ell - \overline{c} > 0. \qquad (\star\star)$$

Wealth inequalities are reduced by savings behavior if savings out of non-accumulated factor income $(1-\widehat{c})\, w_t \ell$ is larger than the (subsistence) consumption flow $\overline{c}$ that is independent

of income and wealth. To see why, we can consider the limit case of an individual with no wealth in equation ($\star$). Such an individual's wealth will increase above $k = 0$ if ($\star\star$) holds, but will otherwise decline further (and become negative: since the model lacks an explicit budget constraint, it cannot address the obvious issue of whether the resulting debt will ever be repaid). The simple derivations above establish that, for similarly mechanical reasons, poor individuals tend to become relatively richer starting from positive wealth levels too.

A.7 Consider the case of the Solow consumption function. Are wealth inequalities increasing or decreasing with time in that model?

In the Solow case, $\overline{c} = 0$, $\widehat{c} = 1 - s$ and $\widetilde{c} = (1 - s)\delta$, so that ($\star\star$) writes $sw_t\ell > 0$, which always hold. Therefore, wealth inequalities are decreasing with time in the Solow configuration.

## PART B : Questions

Are the following statements true, false or uncertain? You should give a structured answer in no more than one page.

B.1 Children are normal goods.

FALSE

(a) A normal good is a good for which demand increases with income. An inferior good is a good whose demand decreases with income.

(b) When one looks at the data, the Malthusian regime (from the beginning of times to the Industrial Revolution) is a period in which any increase in income was followed by an increase in fertility, so that children were looking like normal goods.

(c) Then, the industrial revolution has come with first an increase in income per capita, followed by a decrease in mortality and in fertility. We observe the same phenomenon in developing countries. This is called the demographic transition. This suggest that childen are inferior goods.

(d) In the "Unified Growth" model of the course, we have been able to generate a decrease in fertility even if children were not inferior goods. Because utility was quasi-linear, the demand for chidden had zero income elasticity. As the price of children was the price of food, it is the increase in the relative price of food during the industrial revolution that is in the model responsible for the gradual decrease in fertility.

(e) It is often argued that one should decompose children into quality and quantity components. Quality refers to the level of education. At the same time that the quantity of children has fallen, the quality has increased, such that people are overall spending as much as before on children (perhaps more). Furthermore, In that sense, only the quantity of children is an inferior good.

B.2 The existence of a lower bound for the nominal interest rate is a constraint for monetary policy.

TRUE

(a) Monetary policy aims at price stability, but is also a tool used to smooth business cycles.

(b) Typically, monetary policy is lenient in bad times, so that the nominal interest rate is kept low, which is likely to favour consumption and/or investment in the short run.

(c) Portfolio decision of economic agents: money or bonds.

(d) But when the nominal interest rate is at its lower bound (that is zero ("*zero lower bound*") in a world without risk but that can be negative ("*effective lower bound*") in a risky world), there is no room for a further decrease of the nominal interest rate, so that monetary policy is inefficient. This is the so-called *liquidity trap*. The existence of a lower bound for the nominal interest rate is therefore a constraint for monetary policy.

(e) This was shown in class in two models: IS-LM and the micro-founded model of KRUGMAN.(Here give a brief description of the two models and/or some graphs)

(f) Monetary policy can regain efficiency if it can manage inflation expectations (often referred to as "*forward guidance*")(the real interest can be lowered by lowering the nominal interest rate or by increasing inflation expectations), or by conducting some non-conventional actions (quantitative easing). We haven't formally seen a model of the later.

## PART C : Discussion

Below is an article published in The Economist in April 2019 and entitled "Where growth is concerned, is population destiny?" As always in The Economist, the article is not signed. Read carefully this article before answering the questions below.

C.1 Write a 10 line summary.

- With a large population, Asia was the world's center of gravity before the Industrial Revolution, and is likely to weight half of global production by 2020. Were the two centuries of Western economic domination an anomaly that is about to end? Is population ultimately the main determinant for economics outcome?
- In a world without much international trade, large population means large market and more innovation; hence more growth.
- But a comparison of China and England between 1400 and today shows that population size did not drive economic growth.
- If so, why would the future be different, and why would China and India grow faster because their population is larger?

- Population is not everything. Institutions and culture also matter.
- Furthermore, recent Asian growth has been the consequence of globalization (goods, ideas, people) rather than large population.
- A recent study has shown that eliminating all barriers to migration would raise global welfare threefold.

C.2 Summarize and explain the message of the chart that illustrates that article.

- The chart is showing the evolution of income per capita from 980 to 1850 in China, England, Italy, Holland and India.
- According to the Malthusian regime theory, income per capita growth was low before the industrial revolution, and took off (say) after 1750 in those countries that experienced the industrial revolution (England on that graph). This is what we observe. Chinese and Italian GDP per capita are pretty flat over the period.
- Holland income per capita is an outlier, as it increased continuously from 1350 to 1800. This is mainly explained by the marine trading empire the Dutch build, and that led to the Dutch "Golden Age" between 1600 and 1750. [*This needs not to be known by the students*]
- The chart is also showing that China was slightly richer than England (in per capita terms) until 1400, but that the two economies started diverging after that time. In 1850, England income per capita was about six time the Chinese one.

C.3 According to this article, what are the connections between growth, population size, migrations and international trade?

- It seems that there is not a clear connection between population and growth.
- *Other things equal*, more population means larger market, more innovators, more innovations and more growth.
- But in reality, other things are rarely equal: institutions, bad policies, culture also matter.
- There is no academic consensus on the determinants of long run growth.
- Finally, globalization with transfers of technological know-how, not population size, seems to have triggered Asian recent growth.

C.4 Gross domestic product and gross domestic product per capita are two measures of a country's economic situation. Is one measure more relevant than the other? Why?

- GDP per capita is a good measure of the well-being of a country population. If you had to choose a country in which to be born and receive the average income per capita (and assuming that you only care about income), you would choose a country with high income per capita.

- But it is not a sufficient statistics because it does not capture inequalities of income, but also because it does not fully capture non economic outcomes like literacy, life expectancy, democracy, etc... (although income per capita does correlate with those measures).
- Total GDP of a country is not very relevant as a measure of well-being, but is relevant for the international economic and political influence of a country. Even if GDP per capita in China is 18,000 USD in 2017, and 106,00 in Luxembourg [*those numbers need not to be known by the students*], China is a key player in the global economy while Luxembourg is not.
- Therefore, the two measure are relevant, but for different things.

"Where growth is concerned, is population destiny? "
*The Economist*, Apr 17th 2019

*New research suggests that, in the very long run, size is a great advantage*

For centuries prior to the Industrial Revolution, Asia's massively populous societies made the continent the world's centre of economic gravity. Industrialisation in Europe and North America in the 19th century briefly knocked it from its perch. But now their collective economic might, measured in real output on a purchasing-power-parity basis, is forecast to account for more than half of global production by 2020. Was the West's period of dominance an anomaly, which could only ever have been short-lived? Is population destiny?

It stands to reason that countries with larger populations might enjoy long-run economic advantages. People are the raw material of economic growth, after all. The more there are, the greater the likelihood that one becomes a Gutenberg or a Watt. In a world without much international trade, populous countries offer the largest markets, and comparatively more opportunity to boost economic output through specialisation and trade. Projecting economic growth rates is fantastically hard even over very short time horizons; over centuries, it is as good as impossible. But there are worse strategies than betting on the places with the most people.

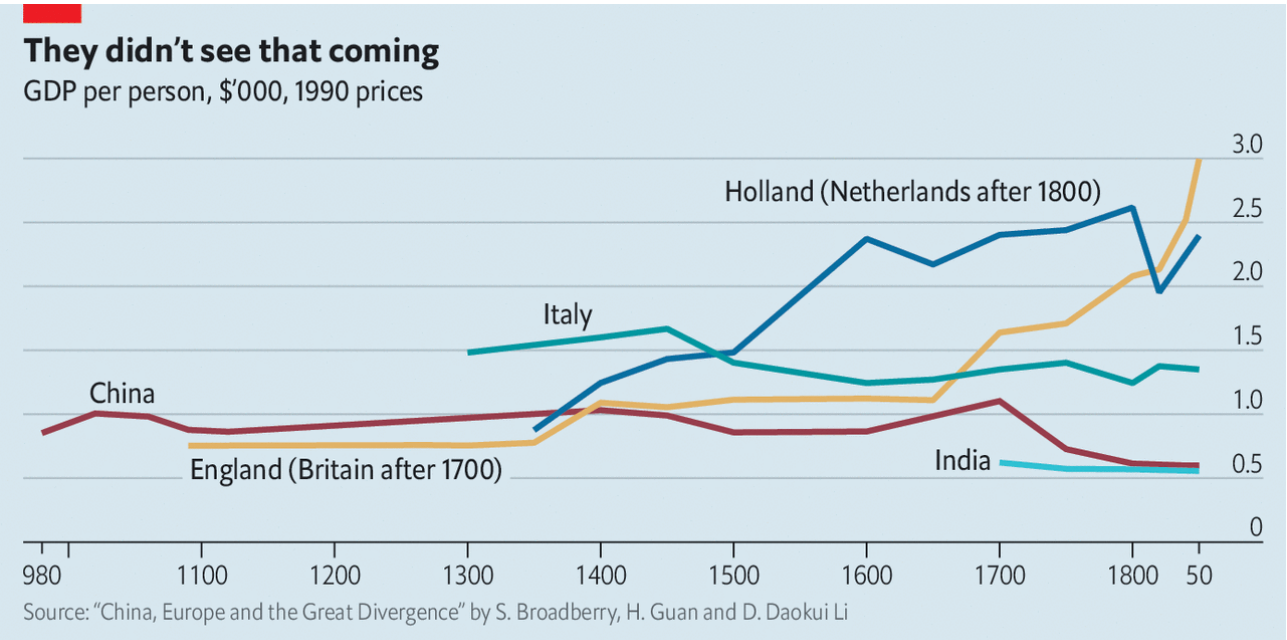
Klaus Desmet of Southern Methodist University, Dávid Krisztián Nagy of CREI, a research institute, and Esteban Rossi-Hansberg of Princeton University do just that. In a paper that last month won them the Robert Lucas prize, which recognises excellent research in political economy, they build a model that yokes economic performance to population size, within which they can run time forward by hundreds of years to watch the balance of economic power change. Long-run growth, they suggest, is driven by improvements in technology. And more populous countries should accumulate more innovation than smaller ones do because the return on developing a new technology is higher–there are more people to buy Edison's light bulb and to enrich Edison, and therefore more incentive to invent the light bulb in the first place.

Leaning against this force, however, is migration. Right now, the richest places are not the most populous. Should it become relatively easy to migrate, people will move from countries that are populous but poor to others that are rich. As migration swells the population of rich places, their long-run dominance is assured because of the link between population size and innovation.

But if there is very little migration, then the populous but poor countries will out-innovate the small but rich ones, and make their way up the income league table. The process is not quick; the authors reckon that convergence takes about 400 years. In practice, rich places tend not to allow much migration from poor ones. That could change, but assuming that it does not, the model delivers a striking forecast: half a millennium from now, Asia and sub-Saharan Africa will have become great engines of productivity.

Stranger things have happened. A millennium ago real output per person was significantly higher in China than in Britain (see chart). To predict that a European backwater would lead the world into the most transformative economic epoch in history would have seemed like madness. Over very long

time horizons the world's poorest places can indeed become the world's richest, even if it does not happen often.




The Economist

Still, if Britain did not have the upper hand over China 1,000 years ago, it did soon after, at least in terms of real output per person. By 1400 incomes in Britain were meaningfully larger than in China (and higher still in the Netherlands and Italy), according to work by Stephen Broadberry of Oxford University, Hanhui Guan of Peking University and David Daokui Li of Tsinghua University. By 1700 the diverging trajectories of China and north-west Europe were clear (though it was anything but obvious just how much further apart they would become). In other words, population over the past millennium has not been destiny. If China's and India's masses did not raise them to prosperity during the past 600 years, what reason is there to believe the future will be different?

**More's the pity**

It is possible that population is destiny, other things equal, but other things are never equal. And so a plague here, or a fateful decision by a Chinese emperor there, can set a region down a path that wipes out the advantages of population. Perhaps those advantages must be harnessed by the right sorts of institutions, or an accommodating culture–which take far longer to develop or adopt than technologies do to emerge. There is no academic consensus regarding what determines economic fortunes over long time horizons, important though the question is. Alternatively, one might argue that conditions have

changed in ways that amplify the power of population. A billion brains seem a more economically potent force in an era of mass education, in contrast to the mass illiteracy that prevailed in the past. But crucially, Asia's recent rise has not been the result of a spurt of indigenous innovation given its impetus by the size of its population. Rather, it has happened as part of a wave of globalisation, which aided the transfer of technological know-how. Openness to exchanges of goods and ideas, or indeed to immigration, is not an immutable parameter, but subject to change based on human preferences. Mr Desmet and his co-authors reckon that eliminating all barriers to migration would raise global welfare threefold – an extraordinary figure that reflects yawning differences in output per person between countries, and the unrealised human potential they represent.

As intriguing as it is to consider the directions in which macro variables such as population or GDP are likely to nudge the world in coming centuries, it is human decisions that will determine which places and people are given the opportunity to become rich. National populations matter to the extent that borders do. It is a depressing notion, but a plausible one, that in half a millennium's time they will matter still.