University College London, 2023-2024
Econ 0107 – Macroeconomics I – Łukasz Rachel & Franck Portier

Midterm Assessment

Answer the four problems. Each problem carries 25% of the total mark.

By submitting this assessment, I pledge my honour that I have not violated UCL's Assessment Regulations which are detailed in the UCL academic manual (chapter 6, section 9 on student academic misconduct procedure), which include (but are not limited to) plagiarism, self-plagiarism, unauthorised collaboration between students, sharing my assessment with another student or third party, access another student's assessment, falsification, contract cheating, and falsification of extenuating circumstances.

I – Uninsurable Health Shocks

Consider an economy with a continuum of households that each solve the following problem

$$\begin{aligned} \max_{\{a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t - b_t\right) \quad &\text{subject to} \\ c_t + a_{t+1} &\leq y + (1+r)\, a_t \\ a_{t+1} &\geq \underline{a}, \end{aligned} \tag{P}$$

where c_t is consumption, a_t is wealth, $\underline{a} \leq 0$ is a borrowing limit, y is non-stochastic income, r is the interest rate, β is a common discount factor satisfying $\beta(1+r) < 1$. $b_t \geq 0$ denotes the necessary expenditures that the household must undertake in order to survive – for example, these could be related to health shocks. Specifically, b_t is a finite-state Markov chain: it takes J discrete values $\{b_1, b_2, ..., b_J\}$ and evolves according to the Markov transition matrix $\mathbf{P}$ with entries $p_{jj'} = \Pr(b_{t+1} = b_{j'} | b_t = b_j)$. For now, take the interest rate r as given. Later r will be determined by an equilibrium condition.

1 – Write the Bellman equation corresponding to problem (P). When doing so, write the expectations operator using the transition probabilities $p_{jj'}$.

2 – Describe an algorithm for solving problem (P). Present the algorithm as numbered steps that one could implement on a computer. Comment briefly on how you would deal with the case where b_t is continuous and follows some known process.

3 – Denote the consumption and saving policy functions by $c(a,b)$ and $a'(a,b)$. For the special case with three income states $J = 3$ and $b_j < y \forall j$ (specific values are not important here, but you can assume that $b_1 = 0$ and the other two are positive), hand-draw two figures of what you expect these policy functions to look like. The figures should have wealth a on the x-axis and different lines for each of the three income states. For the figure displaying the saving policy function $a'(a,b)$, either feature the 45-degree line in the graph or draw it with $a'(a,b) - a$ on the y-axis (this is because we are mostly interested in next period's wealth level a' *relative* to current wealth a). Comment on the main features of your graphs and the economic intuition for why they look like this.

4 – As stated in the description of the economy, there is a large number of households (a continuum). How many Bellman equations do you have to solve? Why?

5 – Describe an algorithm to find the stationary joint distribution $g(a,b)$ of wealth and health. As in subquestion (b), present the algorithm as numbered steps.

6 – We now endogenize the interest rate r. We consider the case of a "Huggett model" in which wealth takes the forms of bonds/IOUs which are in zero net supply. Write down the market clearing condition. Draw a graph of the asset supply curve $A(r)$ and explain how the stationary equilibrium interest rate r is determined.

7 – Suppose that $\underline{a}$ becomes smaller (i.e. larger in absolute value). Explain how this affects the equilibrium condition you have written down, explaining the economic intuition, and comment on whether the stationary equilibrium interest rate stays the same, increases or decreases as a result of this change.

8 – Consider now a production economy with a standard supply side, i.e. perfectly competitive firms operating the production function $Y = K^{\alpha}L^{1-\alpha}$. Write down the market clearing condition. Explain how the stationary equilibrium interest rate r is determined. Do you expect it to be higher or lower than in **6** –? What's the economic intuition?

9 – For this subquestion only, suppose that $b_t = b > 0 \forall t$ (the rest of the economy is as in **8** –). Compute the steady state equilibrium interest rate as a function of parameters in closed form (i.e. analytically).

10 – Implement the algorithm in (b) on the computer. In doing so, parametrize your model in the following way: set $u(c-b) = \frac{(c-b)^{1-\gamma}}{1-\gamma}$, with $\gamma = 2$. Assume that $y = 1$. Set $\beta = 0.95$ and $r = 3\%$ at annual frequency. Finally, assume that b is iid, with $b \in \{0, 0.1, 0.5\}$ with equal probabilities. You may use and adapt a piece of existing code. You can use any programming language you like.

II – Post-Covid Inflation in a Continuous Time NK Model.

This question analyzes a continuous time economy with pricing frictions *à la* Rotenberg. At the end you are asked to think through the post-covid inflation through the lens of the simple model you are about to construct. We have not covered this model in the lectures explicitly, but the question proceeds slowly step-by-step precisely to guide you through this interesting model.

Final consumption good is produced competitively with a constant returns to scale production function

$$Y = \left[\int_0^1 y_j^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}}$$

where y_j is the quantity of intermediate good j. Individual intermediate goods use the production function

$$y_j = l_j$$

where l_j is the quantity of effective labor, which is paid real wage w.

Let $p(t)$ be the nominal price of an individual variety, where we drop the j index. Intermediate firms are ran by entrepreneurs, one firm / entrepreneur per variety. Each entrepreneur chooses a path of $p(t)$ via control $x(t) = \dot{p}(t)/p(t)$ and pays a cost $\frac{\varphi}{2}x(t)^2 Y(t)$ to maximize the value of the firm $Q(t)$:

$$Q(t) = \max_{\{x(\tau)\}_{\tau \geq t}} \int_t^{\infty} e^{-\rho(\tau - t)} \left[\Pi(p(\tau), \tau) - \frac{\varphi}{2}x^2 Y(t)\right] d\tau$$

subject to $\dot{p}(t) = x(t)p(t)$, where $\Pi(p, t)$ are the real profits gross of adjustment cost of a firm charging p at time t.

1 – Denote the aggregate price level with P. Write down the optimisation problem of the final goods producer and derive the demand curve for good j.

2 – Express profits as real revenues minus costs. *Hint: make sure to divide nominal revenues by the price level, P. If you have done things correctly, you'll be able to verify that the derivative of profits is*

$$\frac{\partial \Pi_j}{\partial p_j} = \frac{1}{P} Y \cdot \left(\frac{p_j}{P}\right)^{-\eta} - \eta \left(\frac{p_j}{P} - w\right) Y \cdot \left(\frac{p_j}{P}\right)^{-\eta-1} \frac{1}{P}$$

which will be useful in the next subquestion (you do not actually need to verify this, but you can use it to check if you've answered correctly).

3 – Write down a Hamiltonian for individual firm problem and derive the first order conditions.

4 – Impose symmetry across all firms, i.e. set $p(t) = P(t)$. Show that $x(t) = \pi(t)$. Then show that

$$\dot{\mu} = \rho\mu - \frac{Y}{P}(1 - \eta + \eta w) - \mu\pi.$$

5 – Differentiate the condition $\mathcal{H}_x = 0$ with respect to time, and solve for $\dot{\pi}$, using the expression you derived in (4). To make your New Keynesian Phillips Curve more elegant, define parameter $\kappa := \frac{\eta}{\varphi}$ and let $g_Y := \frac{\dot{Y}}{Y}$. Also define the flexible price inverse markup $w^* := \frac{\eta-1}{\eta}$. Interpret how the sign of $w^* - w(t)$ impacts on $\dot{\pi}$ – and give one sentence intuition for this.

6 – Aside from the entrepreneurs, who are risk neutral and don't play an important role in the economy (except that they own all the firms and consuming all the profits), the economy is populated by measure 1 of households who maximize

$$\int_0^\infty e^{-\rho t} (\log C(t))\, dt$$

subject to $C + \dot{B} = rB + w - T$ where B are the real government bonds issued by the government and T are real taxes. Note that households provide one unit of labor inelastically. Write down the Hamiltonian representing the households' problem and take the first order conditions.

7 – Derive the Euler equation of the households.

8 – Write down the government budget constraint, which states that the change in real government debt outstanding is the difference between the real interest payments on the debt and the taxes collected.

9 – Use the result in **8** – together with the household budget constraint to show that $C(t) = w(t)$. What is g_Y in this economy? Why? Use these results in the Euler equation and the Phillips Curve to derive a system of 2 non-linear ODEs in π and w.

10 – The only part left to specify is the dynamics of the real interest rate $r(t)$. We now close the model with a monetary policy authority, which follows a Taylor rule of the form

$$i(t) = r_0 + \theta_\pi \pi(t).$$

We assume that the zero lower bound does not bind, and that this Taylor rule satisfies the Talor prinicple, $\theta_\pi > 1$. Combine this Taylor rule with the Fisher relation $r(t) = i(t) - \pi(t)$ and use the resulting expression for the real rate of interest in your dynamical system.

11 – Compute the steady state of the system. What is the r_0 in the Taylor rule that is compatible with a zero inflation target?

12 – Assume that the condition on r_0 you derived in **11** – holds. Draw a phase diagram of the system with w on the horizontal axis and π on the vertical axis. Make sure to depict the system dynamics. Is this system saddle path stable? What is the economic intuition for this (e.g. what is the central bank doing here?). You may have seen papers by John Cochrane on this issue.

13 – Now, consider the Covid episode through the lens of the model you constructed. During lockdowns, people realize that "life is short" (although here they are infinitely lived, but let's not worry about this for a second) and they promise to themselves that, at some known point $t' > 0$, when the epidemic is over and vaccines are rolled out, they will "go out and live a little" – i.e. have lavish parties during which they'll spend a lot. In other words, they anticipate a permanent increase in the rate of time preference parameter ρ in period $t' > 0$. On your phase diagram, trace out the dynamics of inflation and real wages from time 0 (when people realize life is short) to t' (when their discount rate rises) and beyond, assuming that r_0 does not change (the central bank follows an unchanged Taylor Rule). First explain how the two curves shift, and then draw in and explain an example path

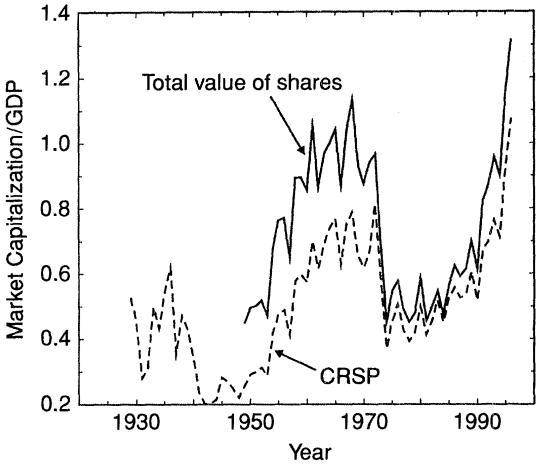
for w and π. What is the intuition for what inflation and wages are at t' and beyond? Specifically, what is the policy of the central bank in this case?

14 – What happens if the central bank is smart and adjusts r_0 so that it equals the new, higher value of ρ? What does this tell you about the importance of the central bank's efforts to figure out what the natural rate of interest is?

III – The Value of the Stock Market.

The goal of this exercice is to use a basic general equilibrium asset pricing model to understand the dynamics of the US stock market from the late 1960's to the late 1990's. Figure 1 shows what happened: a drop from the late 1960's to the late 1980's, then a rapid rebound.

Figure 1: Stock Market/GDP



The proposed explanation is the following. At the end of the 1960's, the markets were expecting Information Technology to arrive in the 1980's. The firms that were public in the late 1960's (the incumbents) were not the ones that would take advantage of IT and were likely to be replaced, hence a decline in their value. Those who will take advantage of IT (the IT innovators) became public (meaning listed on the stock exchange) only in the early 1990's, explaining the rise of the stock market.

Let's write a model, derive its implication for the time path of stock market/GDP and compare to the data. Here is the model.

Time is discrete, and goes from 0 to ∞. The economy is deterministic with a representative infinitely lived agent. A perfectly foreseen consumption stream $\{y_t\}_{t=0}^{\infty}$ yields lifetime utility $\sum_{t=0}^{\infty} \beta^t U(y_t)$.

1 – If consumption is $\{y_t\}_{t=0}^{\infty}$, what is the date-0 price Q_0 of an asset (call it a tree after Lucas) that yields the dividend stream $\{d_t\}_{t=0}^{\infty}$?

Let's assume that there is in this economy a measure one of trees that pay one unit of consumption good each period. These dividends are the only source of consumption good in the economy (no production). The representative household owns the trees and ownership gives right to the dividends.

2 – What is the date-0 price of a tree, denoted P_0?

Assume that in date 0, the following news arrives unexpectedly: out of the measure one of existing trees, a fraction x (the type-I trees) will die at the end of period $T-1$ ($T > 1$), and will be replaced by new infinitely lived better

trees (type-III trees) paying a constant stream of dividends $1+z$, $z>0$. These better trees will not be traded before period T. The remaining $1-x$ fraction of trees (type-II) are unaffected by the news, and will still pay one unit of consumption from 0 to ∞. No other shock than this news will ever hit the economy after period 0.

3 – What is the time path of output (which is also consumption) from 0 to ∞?

4 – What is the time path from 0 to ∞ of the time-t price P_{1t} of a type-I tree?

5 – What is the time path from 0 to ∞ of the time-t price P_{2t} of a type-II tree? It is convenient to write $P_{2t}=P_{1t}+h(t)$, where the function h has to be found.

6 – What is the time path from T to ∞ of the time-t price P_{3t} of a type-III tree? Note that P_{3t} is not defined before T as the property of type-II trees is not traded before T.

7 – The stock market P_t is defined as the time-t market value of public trees. What is the time path of the stock market from 0 to ∞?

8 – To compare to Figure 1, compute P_t/y_t from 0 to ∞ and plot it. Assuming that period 0 is late 1960's and T somewhere in the 1990's, how does the model fit the data?

9 – What are in your opinion the two most crucial and unrealistic assumptions in this analysis?

IV – OLG Model with Production.

Consider an economy in which agents live for two period. Generation t is young in period t and old in period $t+1$. There is a representative agent in each generation. The economy starts in period 0. When young, the agent supplies inelastically $\ell_t=1$ unit of labour and possesses no capital. Wage is w_t. The rental rate of capital is r_t. Labour income is the only source of income for the young, while capital income is the only source of income for the old. Production is done according to the constant returns to scale technology $F(k_t,\ell_t)$. Capital invested in t is productive in $t+1$ and does not depreciate. There is one competitive representative firm. Preferences of generation t is

$$u(c_{t1})+(1+\rho)^{-1}u(c_{2t+1}) \tag{1}$$

We study competitive equilibrium.

1 – Let $s_t=s(w_t,r_t)$ be the saving function of generation t. Using s_t, write the budget constraints of generation t when young and old. Rewrite (1) as $\widetilde{U}(s_t)$

2 – Take first order condition of $\max_{s_t}\widetilde{U}$. Fully differentiate the FOC with respect to s_t, w_t and r_t, and compute the sign of $s_w=\frac{\partial s_t}{\partial w_t}$ and $s_r=\frac{\partial s_t}{\partial r_{t+1}}$. Why is the sign of s_r indeterminate?

3 – From the representative firm FOC, and with the notation $F(k_t,1)=f(k_t)$, express both r_t and w_t as functions of f, f' and k.

4 – Using the results from the previous questions, show that the good market equilibrium condition boils down to an "investment = saving" condition. From there, show that equilibrium capital satisfies a nonlinear recursion of the type $\varphi(k_t,k_{t+1})=0$. Assume a steady state $\overline{k}$ exist. Give a necessary and sufficient condition for local stability.

5 – Assume $u(c)=\log c$ and $F(k,\ell)=k^{\alpha}\ell^{1-\alpha}$. Show that an interior steady state exists, is unique and stable.

End of Exam