Aggregate Precautionary Savings Motives

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21 June 2024

8th AMSE-BdF workshop in Macroeconomics
Marseille

Summary

- Idiosyncratic and aggregate shocks to income.
- Access to a safe bond only
- Debt limit with aggregate and idiosyncratic shocks
- How much of precautionary savings is due to aggregate shocks?

Summary

- How much of precautionary savings is due to aggregate shocks?
- Very much related to the welfare cost of aggregate fluctuations.
  - × If it is small, unlikely to create large precautionary savings.
- Let's review some old literature on this.

- Stochastic consumption

$$c_t = \overline{c} e^{gt} e^{\varepsilon_t}$$

$$\varepsilon_t \sim \mathcal{N}(\mu, \sigma^2)$$

$\mu = -\frac{1}{2}\sigma^2$ s.t. $Ee^{\varepsilon} = 1$

- No shock benchmnark:

$$\widehat{c}_t = \overline{c} e^{gt}$$

- Question: By how much would the representative agent need to be compensated for fluctuations?
- Answer: λ:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\Big((1+\lambda)c_t\Big) = \sum_{t=0}^{\infty} \beta^t u(\widehat{c}_t)$$

Lucas [1987]

λ

$$E_0 \sum_{t=0}^{\infty} \beta^t u\Big((1+\lambda)c_t\Big) = \sum_{t=0}^{\infty} \beta^t u(\widehat{c}_t)$$

- Assume

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

- Then

$$E_0 \sum_{t=0}^{\infty} \beta^t u\Big((1+\lambda)c_t\Big) = \frac{1}{1-\beta e^{g(1-\gamma)}}(1+\lambda)^{1-\gamma}\frac{\overline{c}^{1-\gamma}}{1-\gamma}e^{-\frac{1}{2}\gamma(1-\gamma)\sigma^2}$$

and

$$\sum_{t=0}^{\infty} \beta^t u(\widehat{c}_t) = \frac{1}{1-\beta e^{g(1-\gamma)}}\frac{\overline{c}^{1-\gamma}}{1-\gamma}$$

- which implies

$$1+\lambda = e^{\frac{1}{2}\gamma\sigma^2}$$

Lucas [1987]
λ

$$\lambda \approx \frac{1}{2}\gamma\sigma^2$$

- Assume $\sigma = 5\%$, $\gamma = 5$
- Then

$$\lambda = 0.6\%$$

- The cost of aggregate fluctuations is tiny.
- Aggregate precautionary savings motives is likely to be very small.
- Representative agent, but okay as we are looking at aggregate (non-diversifiable) risk.

Atkeson & Phelan [1994]
Model

- Cost of aggregate fluctuations (and therefore aggregate precautionary savings motive) different with incomplete markets and heterogenous agents.
- "Model":
  - × continuum of agents, all shocks iid.

$$y_{it} = \begin{cases} y^h & \text{with probability} \quad \pi(z_t) \\ y^\ell & \text{with probability} \quad 1 - \pi(z_t) \end{cases}$$

  - × z = aggregate shock.
- Two ways of shutting down aggregate income fluctuations

  Policy 1: Eliminate individual income fluctuation: Replace y_{it} by

$$E_z\left[\pi(z_t)y^h + (1 - \pi(z_t))y^\ell\right]$$

  Policy 2: Eliminate correlation between $\{y_{it}\}$: Replace $\pi(z_t)$ by $E_z\left[\pi(z_t)\right]$

Atkeson & Phelan [1994]
Complete markets

Policy 1: Eliminate individual income fluctuation: Replace y_{it} by
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- Policy 1 increases agent's unconditional welfare.
- Policy 2 has the same impact because perfect risk sharing

Atkeson & Phelan [1994]
Incomplete markets

Policy 1: Eliminate individual income fluctuation: Replace y_{it} by
$$E_z\left[\pi(z_t)y^h + (1-\pi(z_t))y^\ell\right]$$

Policy 2: Eliminate correlation between $\{y_{it}\}$: Replace $\pi(z_t)$ by $E_z\left[\pi(z_t)\right]$

– Assume instead incomplete markets: no assets
  × Policy 1 improves agents utility
  × Policy 2 leaves unconditional individual income distribution unchanged $\rightsquigarrow$ no effect on welfare

Atkeson & Phelan [1994]

Incomplete markets

- Now assume incomplete markets but some asset
- Policy is not Policy 1.
- If asset prices do not change with Policy 2 (reducing correlation between individual risk), it will have no effect on welfare
- Policy 2 can only affect welfare via GE effect and through asset prices (Alvarez & Jermann [2004]).
- Lessons:
  1. if shutting down aggregate risk reduces the correlation of idiosyncratic risk, it may reduce the cost of fluctuations
  2. if it does not, we are back to Lucas $\rightsquigarrow$ not much to expect unless the model has enough non-linearity.

- $\text{income}_{it} = \theta_t \times \theta_{it}^{\sigma(\theta_t)}$
  - $\times$ $\underbrace{\sigma(\theta_t)}_{-}$: countercyclical income dispersion
  - $\times$ shutting down θ_t does affect the correlation of income_{it}, but θ_t is not very variable $\rightsquigarrow$ unlikely to have a big effect
- Borrowing constraint $b_{it} \geq -\phi_{it}(\theta_{it})\overline{\phi}_t(\theta_t)$
- Creates a lot of non-linearity that is counter-cyclical $\rightsquigarrow$ likely to have a large effect on precautionary motives.

Some basic intuition
Lucas

- Assume Lucas' world

$$c_t = \overline{c} e^{\varepsilon_t}$$

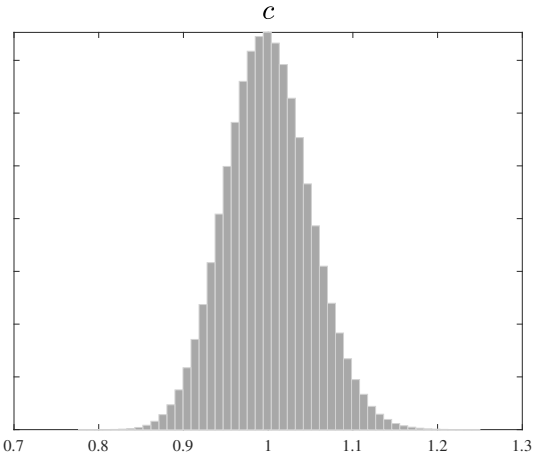
$$\varepsilon_t \sim \mathcal{N}(-\frac{1}{2}\sigma^2, \sigma^2)$$

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

- $\sigma = .05$, $\gamma = 5$, $\overline{c} = 1$,$\beta = .99$

Some basic intuition

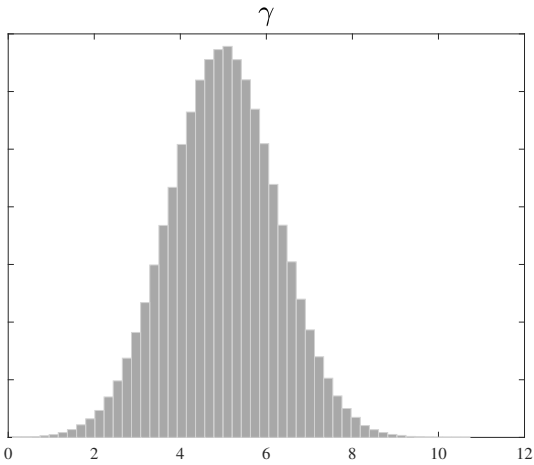
Lucas



- $\lambda = 0.46\%$

Some basic intuition

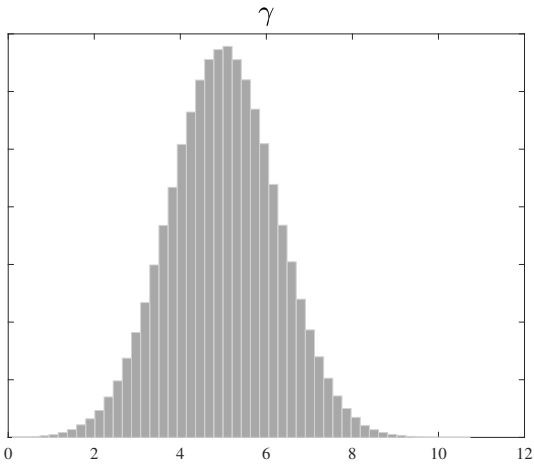
Introduce non-linearity by having risk aversion γ stochastic



- $\lambda = 0.51\%$

Some basic intuition

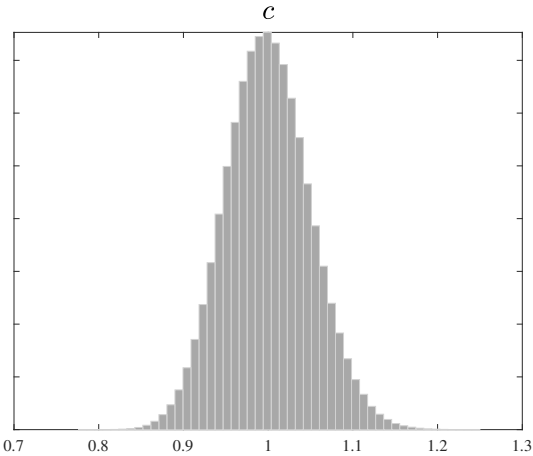
Make γ negatively correlated with c



- $\lambda = 2.24\%$

Some basic intuition

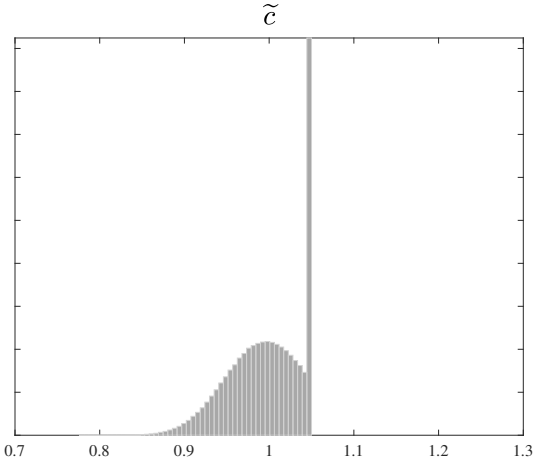
c



- $\lambda = 0.46\%$

Some basic intuition

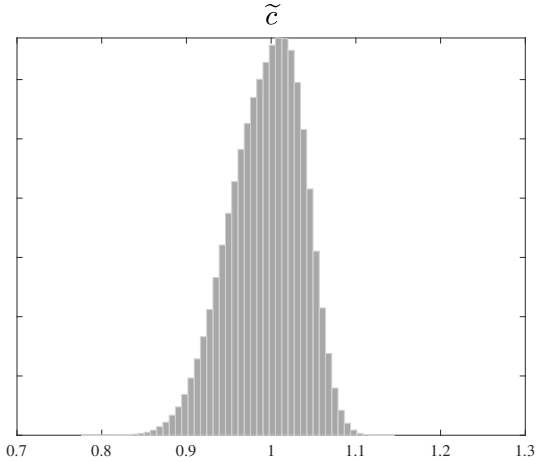
A more standard non-linearity: $\widetilde{c}_t = \min(c_t, 1 + \sigma)$



- $\lambda = 0.77\%$

Some basic intuition

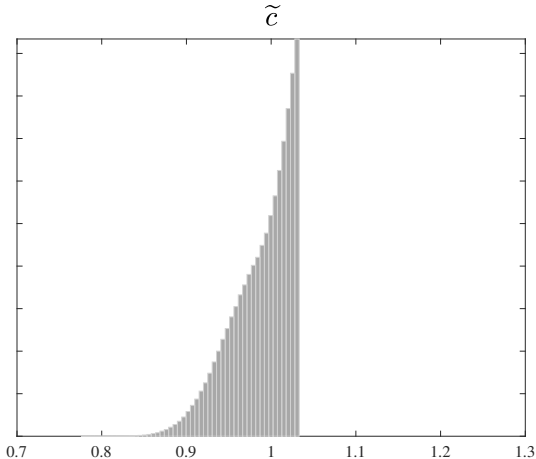
A more standard non-linearity: $\widetilde{c}_t = \min(c_t, 1 + \sigma_t)$, σ_t not correlated to c.



- $\lambda = 0.90\%$

Some basic intuition

A more standard non-linearity: $\widetilde{c}_t = \min(c_t, 1 + \sigma_t)$, σ_t negatively correlated to c.



- $\lambda = 1.36\%$

To conclude

- Fun to read

