AUTUMN TERM MRES ASSESSMENT 2024/25
EXAMINATION COVER SHEET

Module Code:	**ECON0105**
Module Title:	**Macroeconomics**
ASSESSMENT COMPONENT:	**40% EXAM**
ASSESSMENT DATE:	**15th January 2025**
ASSESSMENT START TIME:	**13:30pm**
ASSESSMENT END TIME:	**16:30pm.**
TIME ALLOWED:	**3 hours, closed book.**

INSTRUCTIONS

Write in **blue** or **black ink.**

On the first page of your answer booklet be sure to provide:

- your candidate number
- the number of booklets you submitted.

On each of your answer booklets, make sure to write:

- your candidate number
- the module code

Do not write your name anywhere on the answer sheets;

Do not begin reading the question paper before the start of the exam;

Do check both sides of the exam paper for questions;

Complete all rough work in your answer sheets and cross through any work you do not want marked;

Secure your answer booklets with a treasury tag.

Return the exam paper and your answer booklet to the invigilators. Do not take any of the documents with you when leaving the room.

Raise your hand to alert the invigilator if you require more answer paper, feel unwell or need to take a break (for example to go to the toilet).

University College London, 2024-2025
Econ 0107 – Macroeconomics I – Łukasz Rachel & Franck Portier

Midterm Assessment

Answer the four problems. I and II: 35% of the total mark each. III and IV: 15% of the total mark each.

I – Sustainability of government debt when $R < G$

Consider an economy populated with a continuum of households, a representative firm and a government. We carry our analysis in a detrended model, where each variable that we introduce is already normalized by the balanced growth path level of output. This will become explicit and clear in a moment.

Each household supplies one unit of labour inelastically, and solves the following problem

$$\max_{\{c_t,b_t\}_{t=0}^{\infty}} \int_{t=0}^{\infty} e^{-\rho t}(\log c_t + v(b_t))dt \quad \text{subject to} \\ c_t + \dot{b}_t \leq (R_t - G)b_t + w_t + d_t \tag{P}$$

where ρ is the discount rate, c_t is consumption (relative to balanced growth path level of output), b_t is government debt holdings (also relative to balanced growth path level of output $b_t := \frac{B_t}{Y_t}$), $v(b_t)$ is utility from holding government debt satisfying $v' \geq 0,\ v'' \leq 0$, w_t is the detrended wage rate and d_t is detrended lump sum transfers (or, if negative, taxes).

Competitive representative firm produces using constant returns technology $Y_t = A_t \cdot n_t$, where A_t grows at a constant exponential rate γ and n_t is the labor input. Note that labor market clearing condition implies $n_t = 1 \forall t$. Therefore $Y_t = A_t$, and detrended output is $y_t = 1 \forall t$.

The government sets fiscal and monetary policy. Fiscal policy consists of paths $\{b_t, d_t\}$ subject to the flow budget constraint

$$(R_t - G)b_t + d_t = \dot{b}_t$$

where R_t is the net nominal interest rate, G is the net nominal growth rate of output and d_t is the primary deficit. Monetary policy sets the interest rate equal to the steady state (natural) rate of interest, which results in inflation being equal to the inflation target, $\pi = \pi^*$. Thus, we have $G = \gamma + \pi^*$.

1 – Write the Hamilton-Jacobi-Bellman equation corresponding to problem (P). Take the first order conditions. Provide the economic intuition for each of the conditions. Derive the household Euler Equation.

2 – Comment on what happens to an individual agent's consumption profile over time when the impatience motive is exactly offset by the financial incentive to save (provided by interest rate net of the growth rate).

3 – Combine the government budget constraint and the households' budget constraint to show that the two imply that the resource constraint is satisfied.

4 – For a steady state level of debt b, derive the expression for a natural rate of interest $R^*(b)$. This is the interest rate that holds in a steady state / on the balanced growth path in this economy. Explain the intuition behind this result, both in terms of the level of the natural real interest rate as well as how it changes with b. Can the nominal interest rate be negative?

5 – Using the government budget constraint and the result for $R(b)$, compute the level of primary deficit d that keeps debt constant. Compute the level of debt b^* for which $d(b)$ is maximized.

6 – Now consider for a moment the government budget constraint with constant and exogenous G and R. Draw two phase diagrams, both with d on the vertical and b on the horizontal axis, the left panel assuming $R > G$ and

the right panel assuming $R < G$. Make sure to draw in the arrows for the dynamics of b in both cases. Using your analysis above, explain how, starting in a steady state, a permanent increase in government deficit can be sustained indefinitely when $R < G$, in the sense that it results in a stable level of debt (even without any fiscal consolidation).

7 – Returning to our model, and assuming that $v_b(0) > \rho$ (and thus $R < G$ for b not too large) and $v_b(0) < \infty$, draw the phase diagram. Start by plotting $d(b)$ you derived above, with b on the horizontal and d on the vertical axis. Make sure to draw the arrows indicating debt dynamics. Show (using your diagram) that, starting at some steady state, the government can increase primary deficit and never pay for it (and still stabilize debt in the long-run, albeit at a higher level) as long as the level of debt remains below b^* you derived above. Using that insight, show that such *free lunch* is possible when

$$R(b) < G + bv_{bb} < G.$$

Interpret this finding – in particular, what are the fiscal policy implications of realizing that the natural interest rate is endogenous to the quantity of debt issued?

8 – Suppose that, under political pressure, the central bank raises the inflation target to $\pi^{**} > \pi^*$. Illustrate how this change affects the government debt sustainability analysis in your phase diagram. Does raising the inflation target help the government?

II – Two Agents with Complete Markets

A pure endowment economy consists of two types of consumers. Consumers of type a order consumption streams of the one good according to

$$\sum_{t=0}^{\infty} \beta^t c_t^a.$$

Note that instantaneous utility is linear in c, so that we will have corner solutions (c_t^a can be zero).

Consumers of type b order consumption streams according to

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t^b),$$

$c_t^i \geq 0$ is the consumption of a type i consumer and $\beta \in (0,1)$ is a common discount factor. The consumption good is tradable but nonstorable. There are equal numbers (one) of the two types of consumers.

The consumer of type a receives the following endowment:

$$y_t^a = \mu \quad \forall t \geq 0, \quad \mu > 0,$$

and the consumer of type b receives:

$$y_t^b = \begin{cases} 0 & \text{if } t \geq 0 \text{ is even,} \\ \alpha & \text{if } t \geq 0 \text{ is odd,} \end{cases}$$

where

$$\alpha = \mu \frac{(1+\beta)}{\beta}. \tag{$\mathcal{C}$}$$

Time is discrete and goes from 0 to ∞.

1 – Define a competitive equilibrium with time-0 trading. Be careful to include definitions of all of the objects of which a competitive equilibrium is composed.

2 – Compute a competitive equilibrium allocation with time-0 trading (prices $\{q_t^0\}$ and quantities $\{c_t^a, c_t^b\}$). It is useful to start guessing that c_t^b will be a constant c^b (to be determined) (explain why this is a good guess). From that guess, compute equilibrium prices and quantities starting from the Euler equation of agent b and check that the guess is indeed a correct guess for the obtained equilibrium prices. You can use the normalisation $q_0^0 = 1$. Discuss the results.

3 – Compute the sequence of one-period gross interest rates between t and $t+1$, denoted R_t. Do they differ between odd and even periods?

$\star$

Now consider an economy identical to the preceding one except in one respect. The endowment of consumer a continues to be μ each period, but we assume that the endowment of consumer b is larger (though it continues to be zero in every even period). In particular, we alter the assumption about endowments in condition $(\mathcal{C})$ to the new condition:

$$\alpha > \mu \frac{(1+\beta)}{\beta}. \qquad (\mathcal{C}')$$

We will look for a competitive equilibrium of this economy in which $c_{t \text{ is even}}^a = c_{\text{even}}^a = 0$ and $c_{t \text{ is odd}}^a = c_{\text{odd}}^a > 0 \ \ \forall t$.

4 – Using the optimal behaviour of consumer a, explain why in this equilibrium we must have

$$q_t^0 = \beta^{t-\tau} q_\tau^0 \text{ for } t > \tau \text{ and } t, \tau \text{ odd.}$$

or equivalently

$$q_t^0 = \beta^{t-1} q_1^0 \quad \text{for } t \text{ odd.} \qquad (\star)$$

5 – Find the competitive equilibrium allocation and price system for this economy (use $(\star)$, the Euler equation of consumer b between period t even and period 0 and the ibc of consumer b to find c_{odd}^b, and from there prices and quantities).

6 – Compute the sequence of one-period interest rates implicit in the equilibrium price system that you computed in question **5**. Are interest rates higher or lower than those you computed in question **2**?

III – Aiyagari and Inequality

1 – Write down the Bellman equation of a consumer in the Bewley-Hugget-Aiyagari type of model with a two state income process and no aggregate uncertainty.

2 – Explain in words why in the Aiyagari model, if there is aggregate uncertainty, the wealth distribution is one of the relevant state variables.

3 – When calibrated in a standard fashion, the Aiyagari model fails to replicate the large amount of wealth concentration at the top of the wealth distribution in the United States, United Kingdom and other developed countries. In particular it fails to match statistics like the top 1% and top 0.1% wealth shares and the observed Pareto tail of empirical wealth distributions. List three extensions of the Aiyagari model that researchers have proposed to explain top wealth inequality, and in each case provide a one-sentence intuition for the proposed mechanism.

IV – Optimal Taxation with Commitment

Consider the environment of Lecture 5 of Franck Portier (infinite horizon, deterministic environment, complete markets, one representative household and one representative firm with Neoclassical production function and labour and capital as inputs, a gouvernment that spends an exogenous stream and taxes using labour income tax, capital income tax and issues one period real debt).

1 – Explain the logic of the primal and dual approach to fin the optimal path of taxes.

2 – Explain the result of zero capital tax at the steady state.