Nonlinear Phillips Curves (?) with $\log\theta$ vs. θ

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Following a suggestion of Pascal Michaillat, this note reproduces Figures 1 and 2 of Beaudry, Hou and Portier, "Did Recent Inflation Reflect a Nonlinear Phillips Curve?" (*AER: Insights*, 2027, forthcoming). Each figure is then repeated with labour market tightness measured as $\theta = V/U$ instead of $\log\theta$.

Section 2 repeats all figures of Section 1 with the cubic fit replaced by a kinked linear one, whose slope can change at $\log\theta = 0$ (equivalently $\theta = 1$).

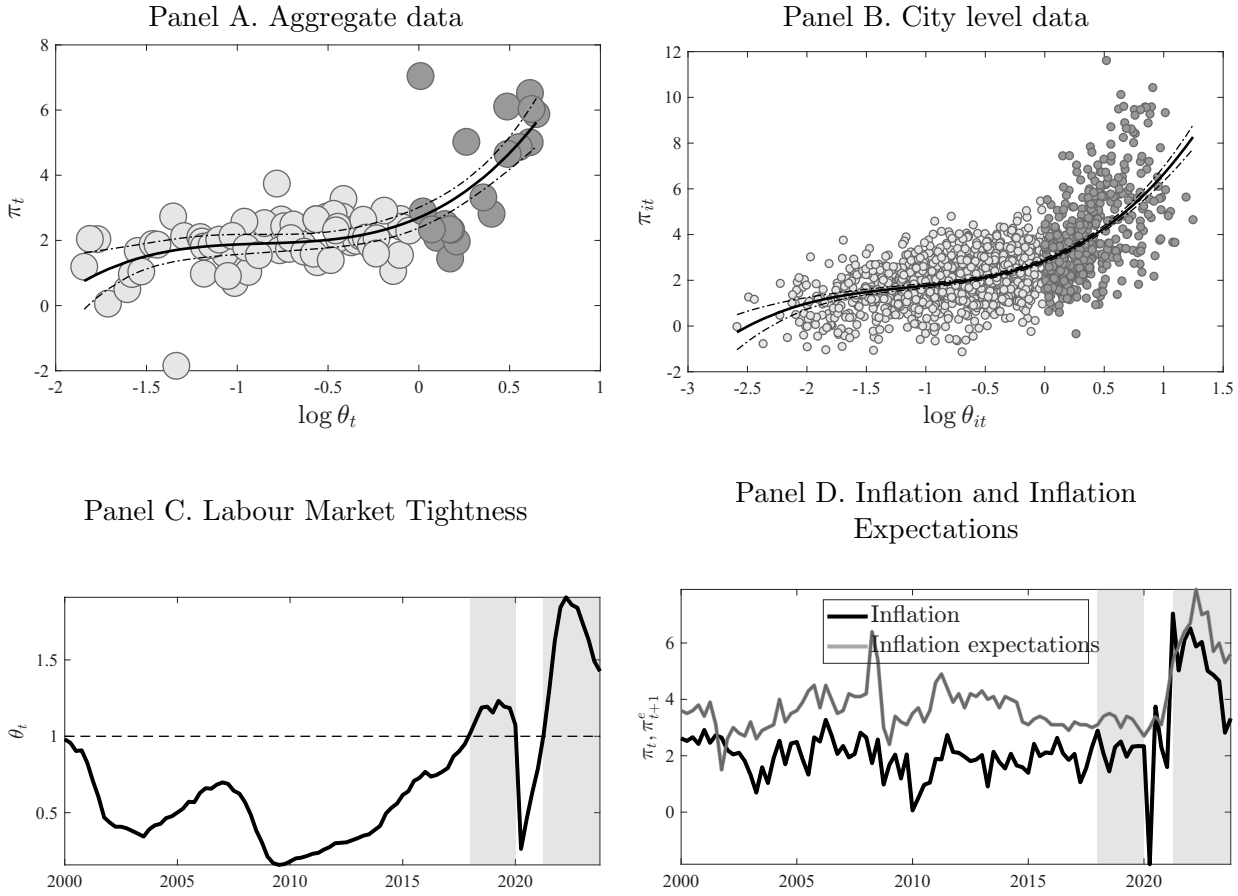
1 Cubic Relation

Here we fit the data with a relation

$$y = a + b_1\, x + b_2\, x^2 + b_3\, x^3$$

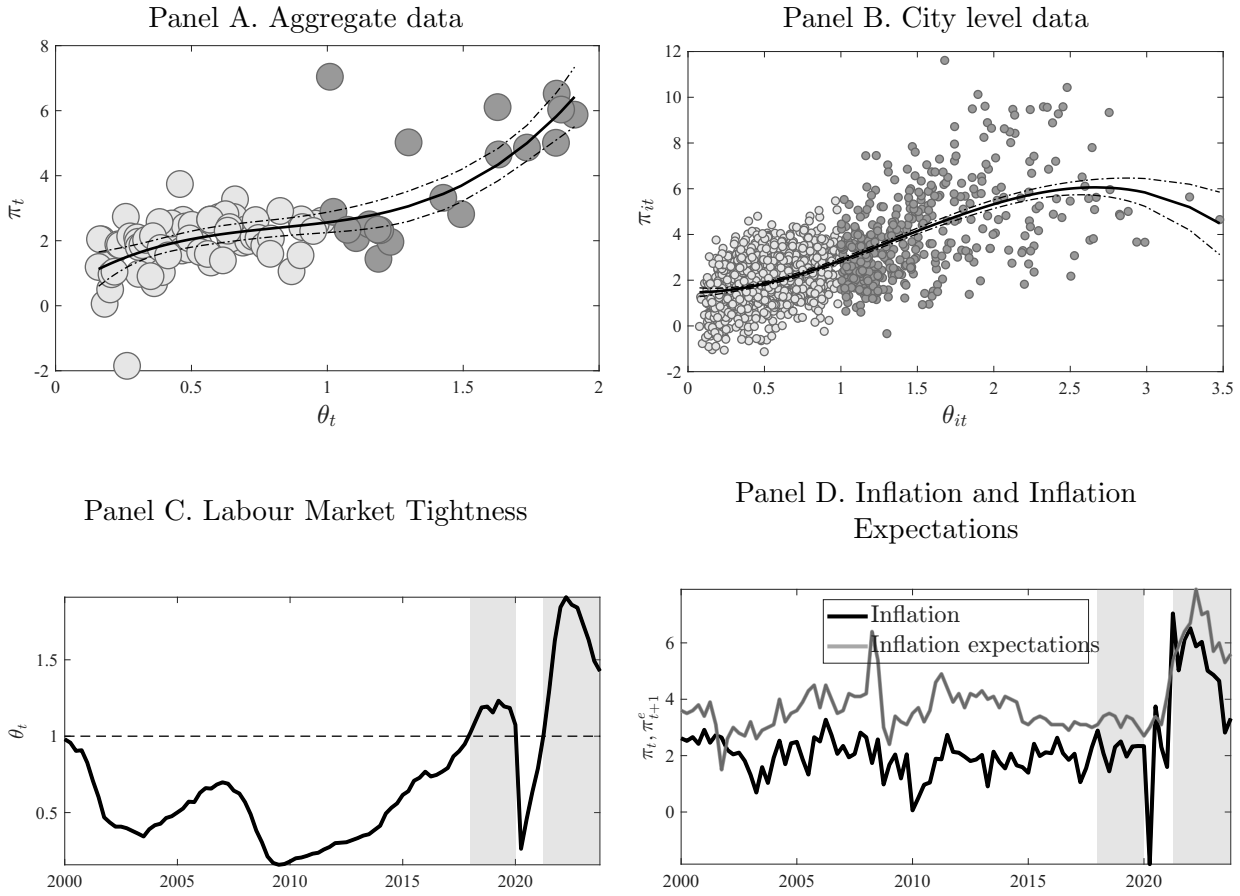
and plot it in Figures 1 and 3 (part (a), $x = \log\theta$) and Figures 2 and 4 (part (b), $x = \theta$). The dependent variable y is π (Figures 1 and 2), residualized π (Figures 3 and 4, Panels A and B), residualized $\pi - .99\,\pi^e$ (Figures 3 and 4, Panel C) or $\pi - .99\,\pi^e$ (Figures 3 and 4, Panel D).

Figure 1: Labour Market Tightness, Inflation and Inflation Expectations, Raw Data ($\log \theta$, as in the paper)



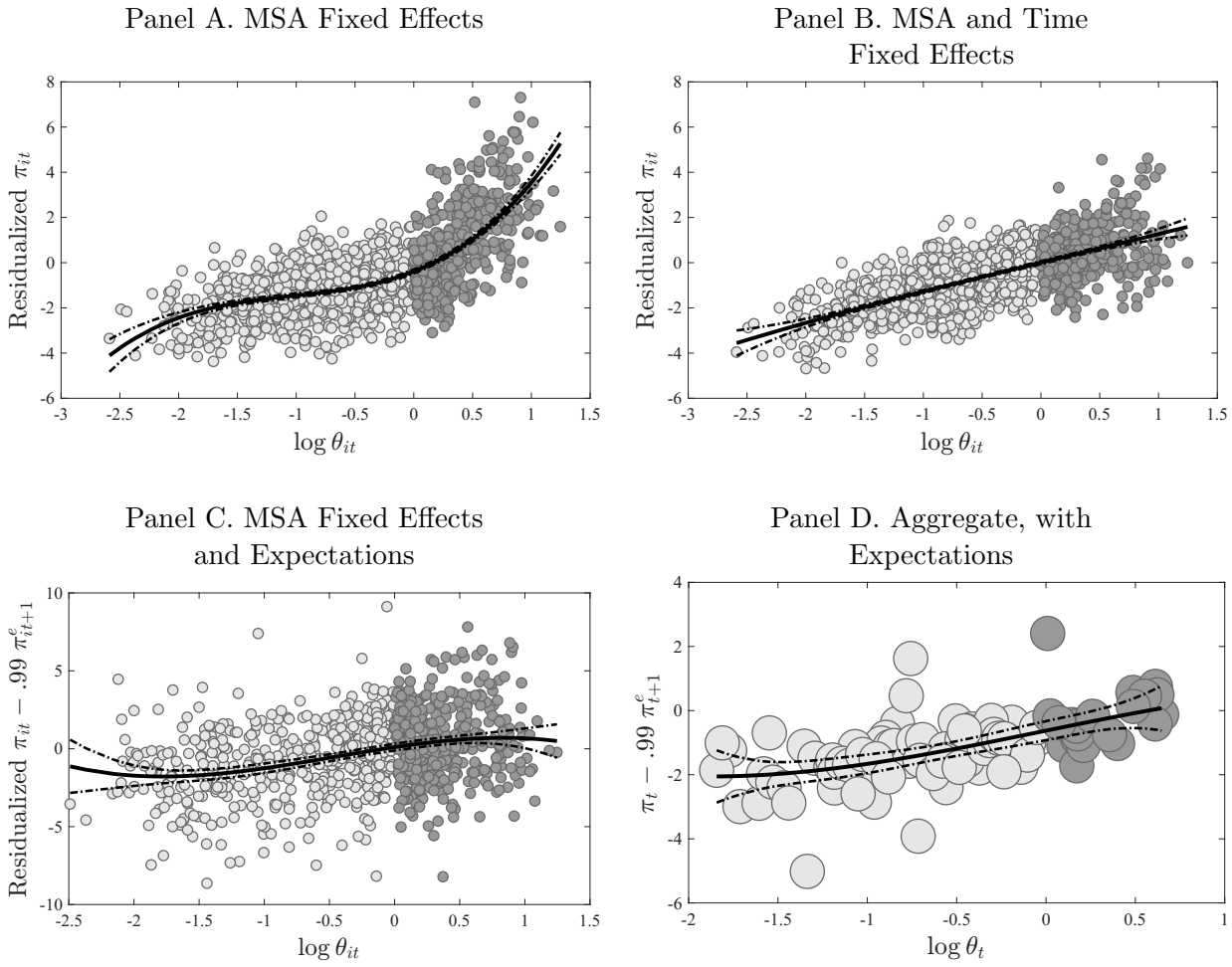
Notes: Each dot represents a quarter (Panel A) or a quarter-city (Panel B). Labour market tightness is measured as $\log \theta$, where θ is the ratio of job vacancies to unemployment (V/U). Inflation is quarter-to-quarter (Panels A and D) or year-to-year (Panel B) CPI core inflation (annualized). For Panels A and B, light gray dots correspond to $\log \theta \leq 0$, dark gray dots correspond to $\log \theta > 0$ and the black line is the fitted cubic relation between π and $\log \theta$, dotted lines delimit the 95% confidence interval. Expectations are mean one-year-ahead and obtained from the Michigan Survey of Consumers. Grey areas represent quarters with $\theta > 1$. Sample is 2000Q1-2023Q4 for Panels A, C and D and 2001Q4-2024Q3 for Panel B.

Figure 2: Labour Market Tightness, Inflation and Inflation Expectations, Raw Data (θ instead of $\log\theta$)



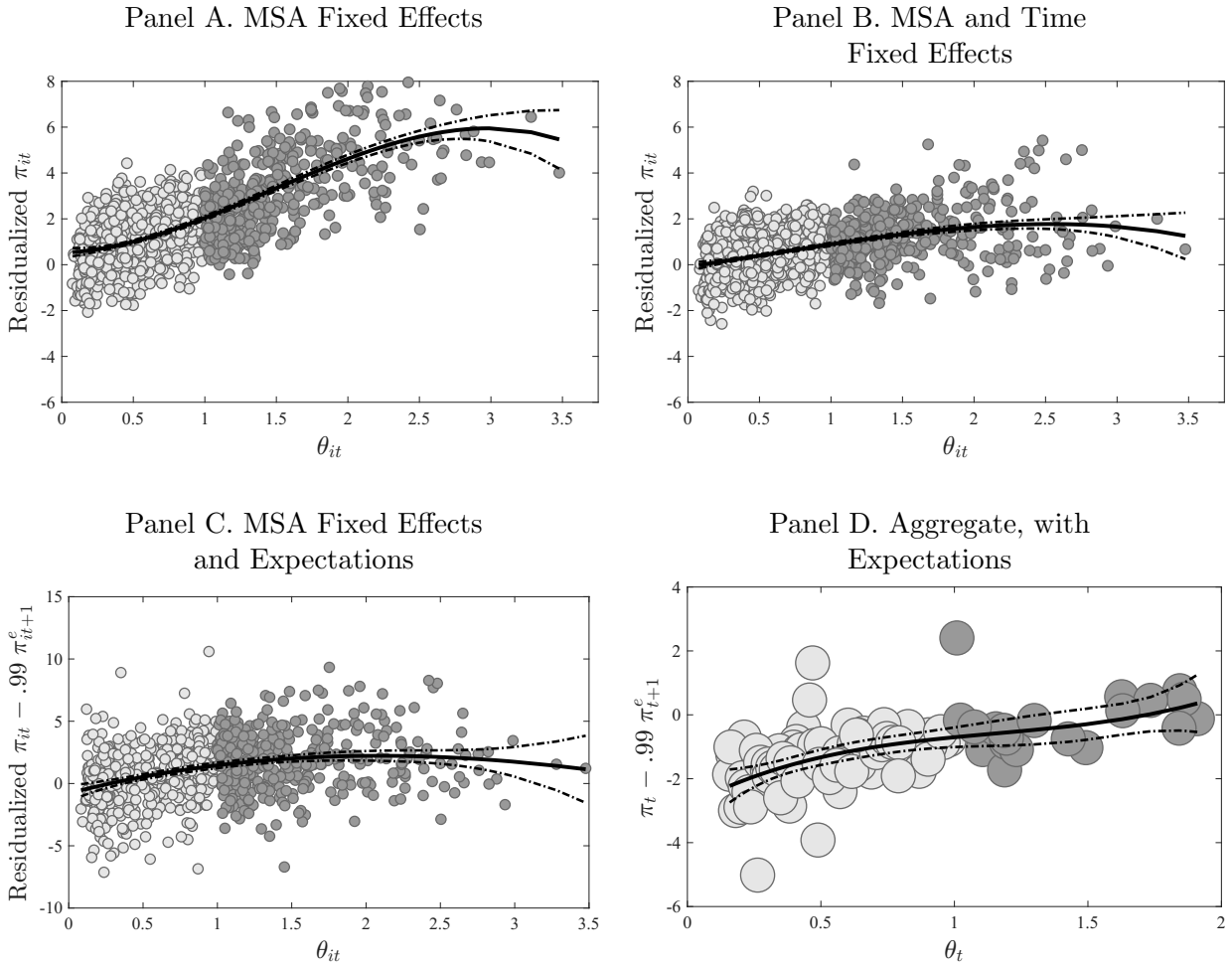
Notes: Same as Figure 1, with θ instead of $\log\theta$ on the horizontal axis of Panels A and B, and the fitted cubic is in θ. Light gray dots correspond to $\theta \leq 1$, dark gray dots to $\theta > 1$. Panels C and D are identical to Figure 1.

Figure 3: Inflation and Labour Market Tightness, City Level Data and Aggregate Data ($\log\theta$, as in the paper)



Notes: Each dot represents a quarter-city in Panels A, B and C, and a quarter in Panel D. Dark dots indicate observations with $\log\theta > 0$ and light dots observations with $\log\theta \leq 0$. Inflation is quarter-to-quarter (Panel C and D) or year-to-year (Panels A and B) CPI core inflation (annualized). In Panel A, residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} = \alpha_i + \kappa \log\theta_{it} + \varepsilon_{it}$ and computed as $\pi_{it} - \alpha_i$. In Panel B, residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} = \alpha_i + \gamma_t + \kappa \log\theta_{it} + \varepsilon_{it}$ and is computed as $\pi_{it} - \alpha_i - \gamma_t$. In Panel C, residualized inflation is obtained from the two-way linear fixed effects regression $\pi_{it} - .99\ \pi^e_{it+1} = \alpha_i + \kappa \log\theta_{it} + \varepsilon_{it}$ and computed as $\pi_{it} - .99\pi^e_{it+1} - \alpha_i$. The measure of π^e_{it+1} is obtained from Binder, Kamdar and Ryngaert (2024). In Panel D, expectations are median one-year-ahead and obtained from the Michigan Survey of Consumers. Both expectations are adjusted to proxy one-quarter-ahead expectations. In all panels, the black line is the fitted cubic relation between (residualized) π and $\log\theta$, dotted lines delimit the 95% confidence interval. Sample is 2001Q4-2024Q3 for Panels A, B and C and 2000Q1-2023Q4 for Panel D.

Figure 4: Inflation and Labour Market Tightness, City Level Data and Aggregate Data (θ instead of $\log\theta$)



Notes: Same as Figure 3, with θ replacing $\log\theta$ everywhere. Residualized inflation comes from the regressions $\pi_{it} = \alpha_i + \kappa\,\theta_{it} + \varepsilon_{it}$ (Panel A), $\pi_{it} = \alpha_i + \gamma_t + \kappa\,\theta_{it} + \varepsilon_{it}$ (Panel B) and $\pi_{it} - .99\,\pi^e_{it+1} = \alpha_i + \kappa\,\theta_{it} + \varepsilon_{it}$ (Panel C). The black line is the fitted cubic relation between (residualized) π and θ. Dark dots indicate $\theta > 1$, light dots $\theta \leq 1$.

Table 1: Parameter Estimates of the Cubic Relation

(a) $x = \log\theta$						
	Figure 1		Figure 3			
	Panel A	Panel B	Panel A	Panel B	Panel C	Panel D
a	2.70^{***}	2.87^{***}	-0.39^{***}	0.00	0.10	-0.63^{***}
	(0.16)	(0.05)	(0.05)	(0.04)	(0.11)	(0.15)
b_1	2.41^{***}	2.05^{***}	2.03^{***}	1.26^{***}	1.24^{***}	1.14^{***}
	(0.28)	(0.07)	(0.07)	(0.05)	(0.18)	(0.26)
b_2	2.60^{***}	1.34^{***}	1.44^{***}	-0.01	-0.39^{*}	-0.01
	(0.57)	(0.11)	(0.10)	(0.08)	(0.24)	(0.54)
b_3	1.01^{***}	0.39^{***}	0.47^{***}	0.01	-0.28^{**}	-0.11
	(0.31)	(0.06)	(0.05)	(0.04)	(0.14)	(0.29)
Obs.	96	1492	1492	1492	842	96
R^2	0.55	0.46	0.50	0.52	0.11	0.34
(b) $x = \theta$						
	Figure 2		Figure 4			
	Panel A	Panel B	Panel A	Panel B	Panel C	Panel D
a	0.35	1.47^{***}	0.52^{***}	-0.09	-0.79^{**}	-2.84^{***}
	(0.55)	(0.12)	(0.12)	(0.09)	(0.31)	(0.53)
b_1	5.79^{**}	-0.15	0.21	0.92^{***}	3.06^{***}	4.35^{*}
	(2.31)	(0.41)	(0.38)	(0.30)	(0.95)	(2.26)
b_2	-6.01^{**}	2.05^{***}	1.69^{***}	0.14	-0.86	-3.11
	(2.74)	(0.35)	(0.33)	(0.26)	(0.78)	(2.68)
b_3	2.43^{**}	-0.50^{***}	-0.39^{***}	-0.08	0.04	0.89
	(0.93)	(0.08)	(0.08)	(0.06)	(0.18)	(0.90)
Obs.	96	1492	1492	1492	842	96
R^2	0.58	0.47	0.53	0.21	0.11	0.34

Notes: OLS estimates of the fitted cubic relation $y = a + b_1\,x + b_2\,x^2 + b_3\,x^3$ plotted in Figures 1 and 3 (part (a), $x = \log\theta$) and Figures 2 and 4 (part (b), $x = \theta$). The dependent variable y is π (Figures 1 and 2), residualized π (Figures 3 and 4, Panels A and B), residualized $\pi - .99\,\pi^e$ (Figures 3 and 4, Panel C) or $\pi - .99\,\pi^e$ (Figures 3 and 4, Panel D). Standard errors in parentheses; ***, ** and * denote significance at the 1%, 5% and 10% levels (two-sided t test). Standard errors and significance levels do not account for the first-stage fixed-effects regressions used to residualize the data in Panels A–C of Figures 3 and 4.

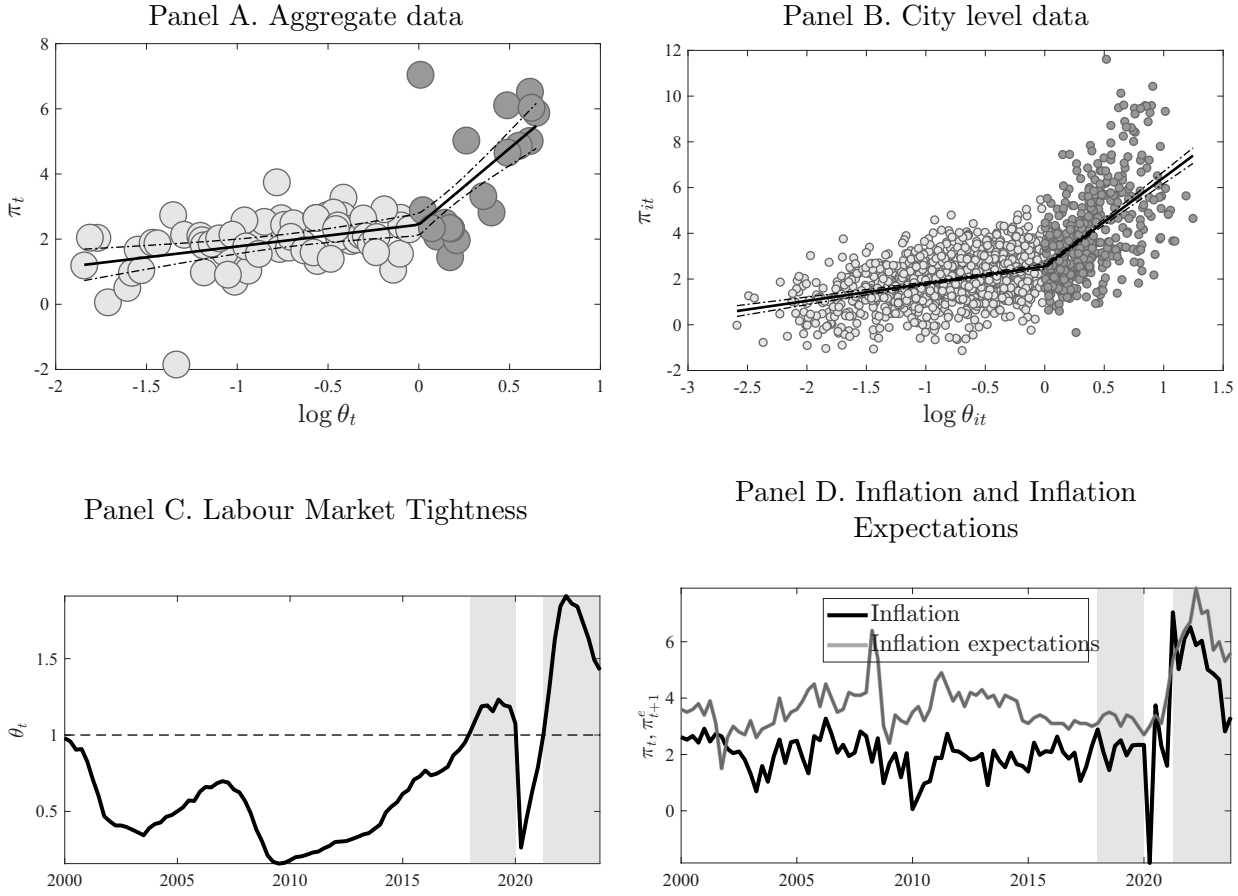
2 Kinked Linear Relation

Here we fit the data with a relation

$$y = a + b\,x + c\,\max(x - \overline{x}, 0)$$

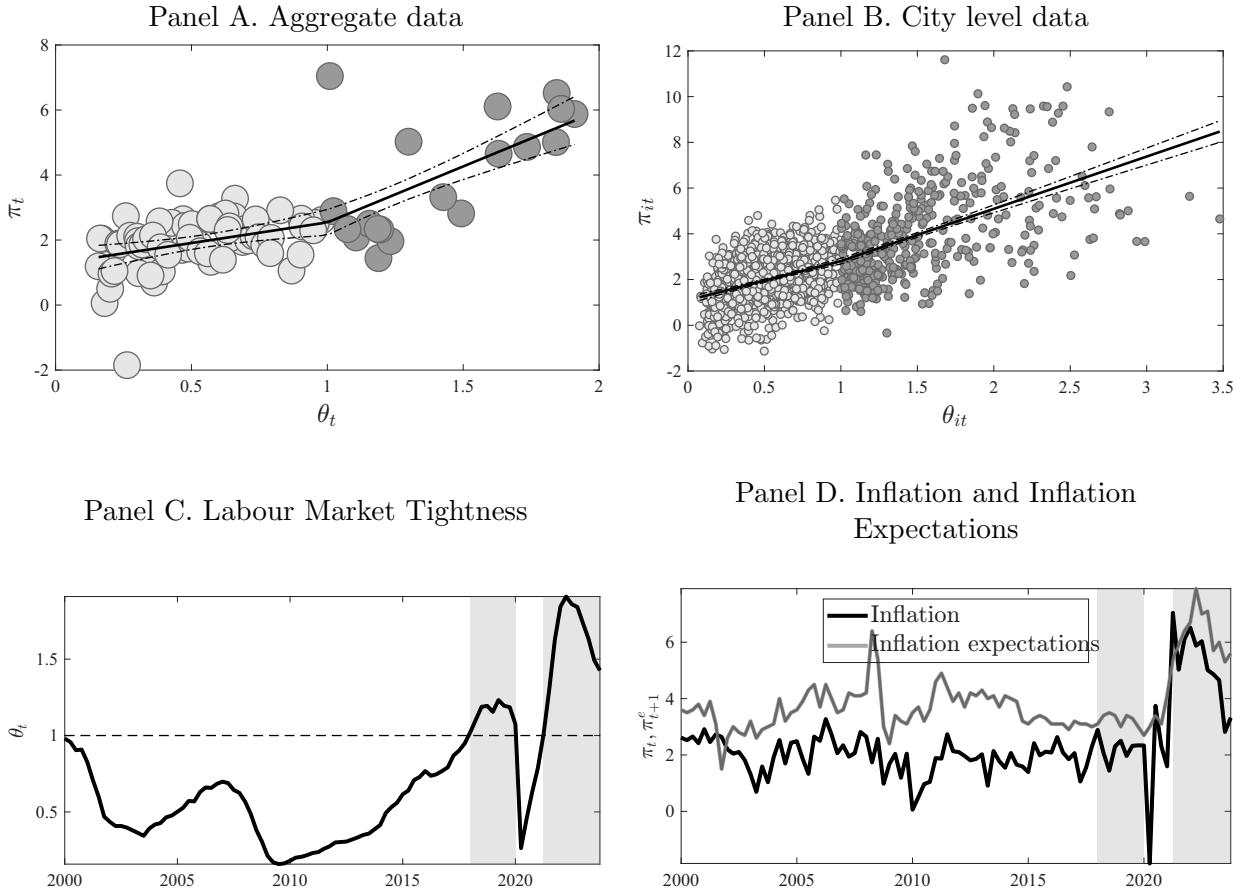
and plot it in Figures 5 and 7 (part (a), $x = \log\theta$, $\overline{x} = 0$) and Figures 6 and 8 (part (b), $x = \theta$, $\overline{x} = 1$). The dependent variable y is π (Figures 5 and 6), residualized π (Figures 7 and 8, Panels A and B), residualized $\pi - .99\,\pi^e$ (Figures 7 and 8, Panel C) or $\pi - .99\,\pi^e$ (Figures 7 and 8, Panel D).

Figure 5: Labour Market Tightness, Inflation and Inflation Expectations, Raw Data ($\log \theta$, kinked linear fit)



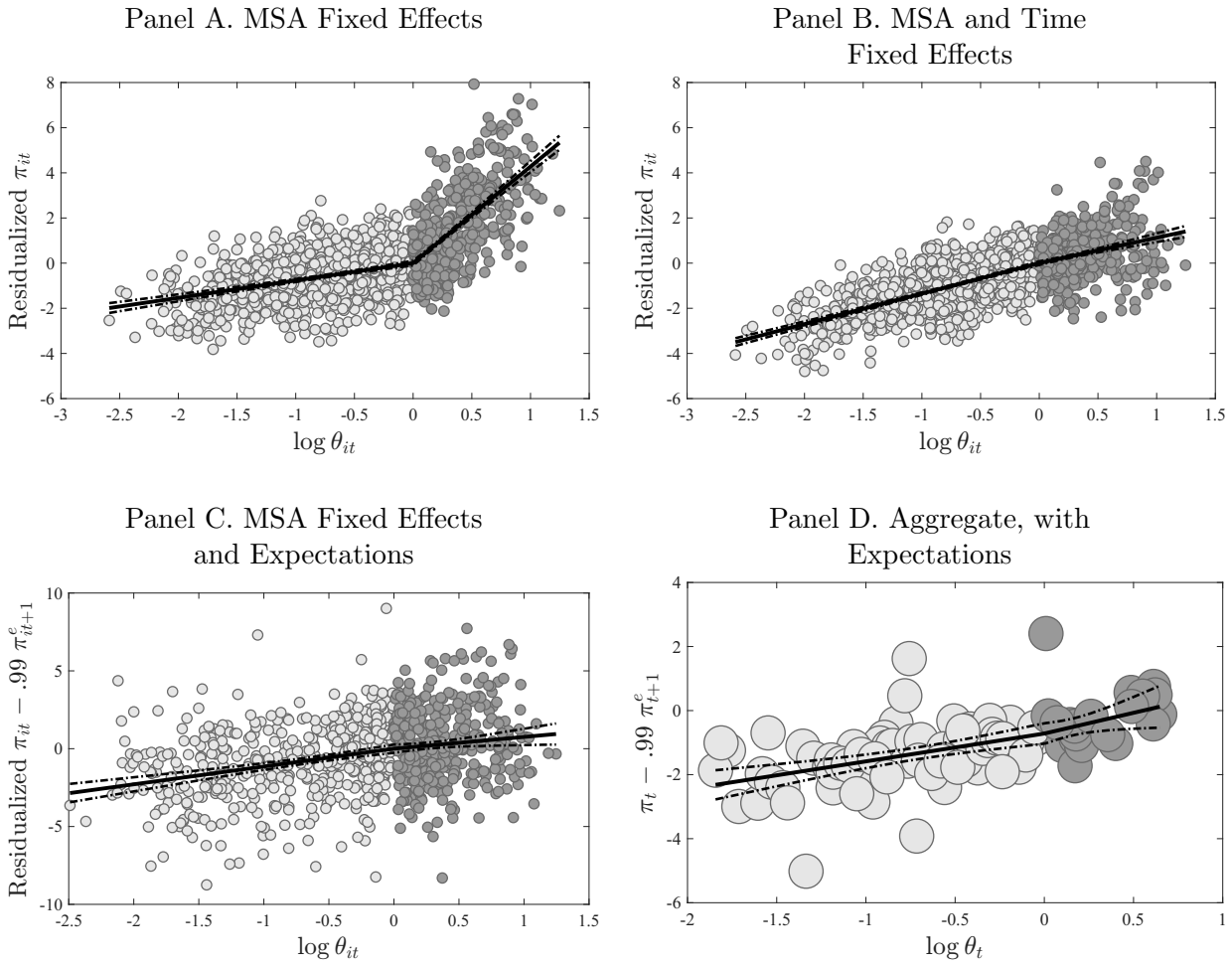
Notes: Same as Figure 1, except that the black line is the fitted kinked linear relation $\pi = a + b \log \theta + c \max(\log \theta, 0)$, so that the slope is b for $\log \theta \leq 0$ and $b + c$ for $\log \theta > 0$. Dotted lines delimit the 95% confidence interval.

Figure 6: Labour Market Tightness, Inflation and Inflation Expectations, Raw Data (θ, kinked linear fit)



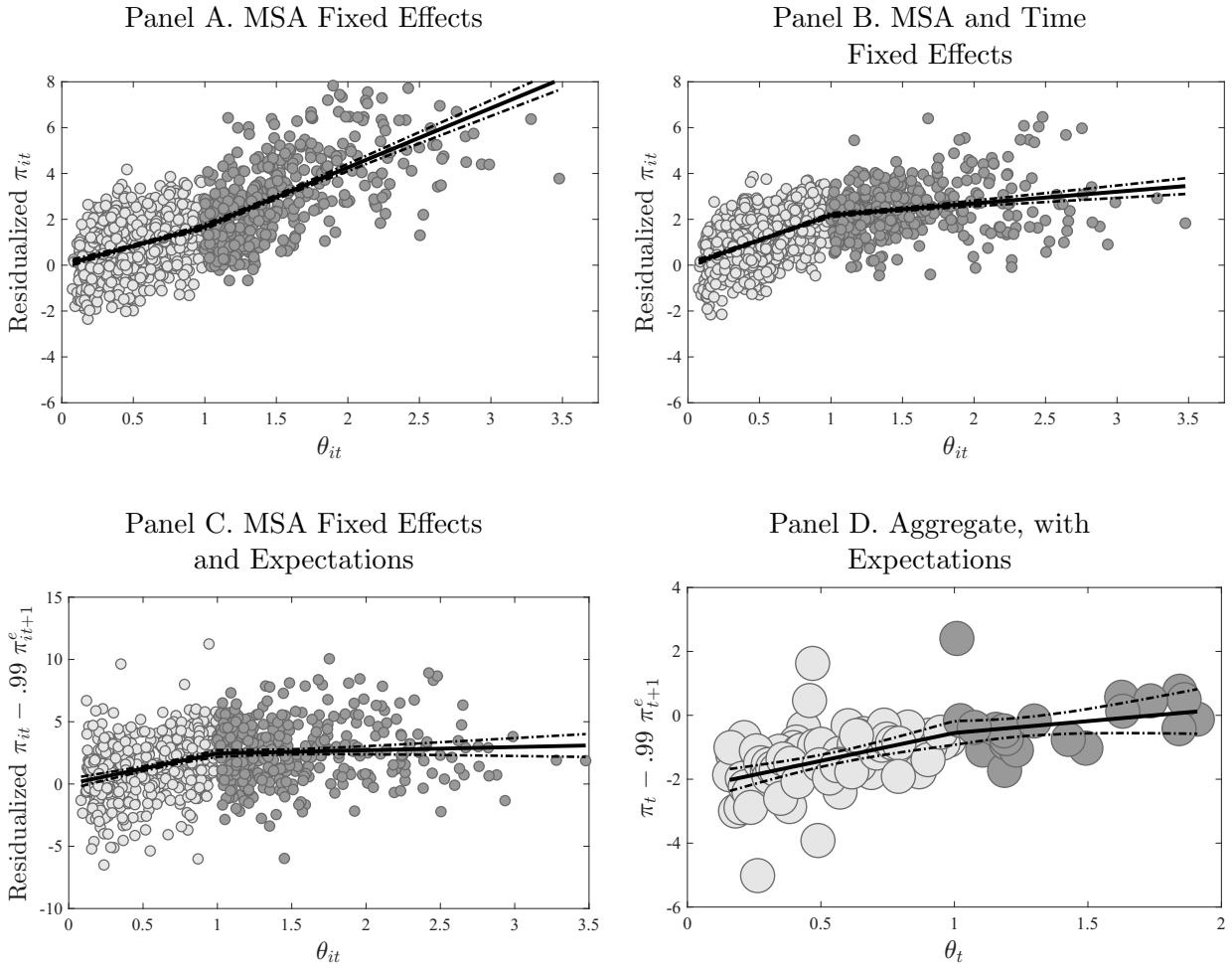
Notes: Same as Figure 2, except that the black line is the fitted kinked linear relation $\pi = a + b\,\theta + c\,\max(\theta - 1, 0)$, so that the slope is b for $\theta \leq 1$ and $b + c$ for $\theta > 1$. Dotted lines delimit the 95% confidence interval.

Figure 7: Inflation and Labour Market Tightness, City Level Data and Aggregate Data ($\log\theta$, kinked linear fit)



Notes: Same as Figure 3, except that the kinked linear model replaces the cubic both in residualizing the data and in the fitted line. Residualized inflation is obtained from the regressions $\pi_{it} = \alpha_i + \kappa_1 \log\theta_{it} + \kappa_2 \max(\log\theta_{it}, 0) + \varepsilon_{it}$ (Panel A), $\pi_{it} = \alpha_i + \gamma_t + \kappa_1 \log\theta_{it} + \kappa_2 \max(\log\theta_{it}, 0) + \varepsilon_{it}$ (Panel B) and $\pi_{it} - .99\ \pi^e_{it+1} = \alpha_i + \kappa_1 \log\theta_{it} + \kappa_2 \max(\log\theta_{it}, 0) + \varepsilon_{it}$ (Panel C), and computed as $\pi_{it} - \alpha_i$ (Panels A and C, with π^e subtracted in C) or $\pi_{it} - \alpha_i - \gamma_t$ (Panel B). In all panels, the black line is the fitted relation $a + b\log\theta + c\max(\log\theta, 0)$ between (residualized) π and $\log\theta$, with slope b for $\log\theta \leq 0$ and $b + c$ for $\log\theta > 0$; dotted lines delimit the 95% confidence interval.

Figure 8: Inflation and Labour Market Tightness, City Level Data and Aggregate Data (θ, kinked linear fit)



Notes: Same as Figure 4, except that the kinked linear model replaces the cubic both in residualizing the data and in the fitted line. Residualized inflation is obtained from the regressions $\pi_{it} = \alpha_i + \kappa_1\theta_{it} + \kappa_2 \max(\theta_{it} - 1, 0) + \varepsilon_{it}$ (Panel A), $\pi_{it} = \alpha_i + \gamma_t + \kappa_1\theta_{it} + \kappa_2 \max(\theta_{it} - 1, 0) + \varepsilon_{it}$ (Panel B) and $\pi_{it} - .99\ \pi^e_{it+1} = \alpha_i + \kappa_1\theta_{it} + \kappa_2 \max(\theta_{it} - 1, 0) + \varepsilon_{it}$ (Panel C), and computed as $\pi_{it} - \alpha_i$ (Panels A and C, with π^e subtracted in C) or $\pi_{it} - \alpha_i - \gamma_t$ (Panel B). In all panels, the black line is the fitted relation $a + b\,\theta + c \max(\theta - 1, 0)$ between (residualized) π and θ, with slope b for $\theta \leq 1$ and $b + c$ for $\theta > 1$; dotted lines delimit the 95% confidence interval.

Table 2: Parameter Estimates of the Kinked Linear Relation

(a) $x = \log\theta$						
	Figure 5		Figure 7			
	Panel A	Panel B	Panel A	Panel B	Panel C	Panel D
a	2.44***	2.56***	0.00	0.00	0.00	−0.72***
	(0.17)	(0.06)	(0.05)	(0.04)	(0.13)	(0.16)
b	0.67***	0.76***	0.77***	1.35***	1.14***	0.87***
	(0.20)	(0.06)	(0.06)	(0.05)	(0.15)	(0.19)
c	4.02***	3.13***	3.51***	−0.23	−0.39	0.43
	(0.76)	(0.20)	(0.18)	(0.15)	(0.43)	(0.72)
$b + c$	4.69***	3.88***	4.28***	1.13***	0.76**	1.29**
	(0.65)	(0.16)	(0.15)	(0.12)	(0.34)	(0.61)
Obs.	96	1492	1492	1492	842	96
R^2	0.55	0.47	0.55	0.52	0.11	0.33
(b) $x = \theta$						
	Figure 6		Figure 8			
	Panel A	Panel B	Panel A	Panel B	Panel C	Panel D
a	1.27***	1.09***	0.00	0.00	0.00	−2.30***
	(0.24)	(0.08)	(0.07)	(0.06)	(0.22)	(0.22)
b	1.27***	1.71***	1.68***	2.18***	2.46***	1.75***
	(0.39)	(0.12)	(0.12)	(0.09)	(0.30)	(0.37)
c	2.17***	0.57***	0.91***	−1.67***	−2.20***	−1.01
	(0.79)	(0.21)	(0.19)	(0.15)	(0.45)	(0.74)
$b + c$	3.44***	2.29***	2.59***	0.51***	0.26	0.74
	(0.51)	(0.11)	(0.10)	(0.08)	(0.22)	(0.48)
Obs.	96	1492	1492	1492	842	96
R^2	0.55	0.46	0.53	0.42	0.11	0.33

Notes: OLS estimates of the fitted kinked linear relation $y = a + b\,x + c\,\max(x - \overline{x}, 0)$, with threshold $\overline{x} = 0$ for $x = \log\theta$ (part (a), Figures 5 and 7) and $\overline{x} = 1$ for $x = \theta$ (part (b), Figures 6 and 8). The slope is b below the threshold and $b + c$ above it. The dependent variable y is defined as in Table 1, with residualization based on the kinked linear model. Standard errors in parentheses; ***, ** and * denote significance at the 1%, 5% and 10% levels (two-sided t test). Standard errors and significance levels do not account for the first-stage fixed-effects regressions used to residualize the data in Panels A–C of Figures 7 and 8.